Empirical Methods for the Analysis of the Energy Transition

Day 2

Prof. Mar Reguant

Summer School 2024

Barcelona School of Economics

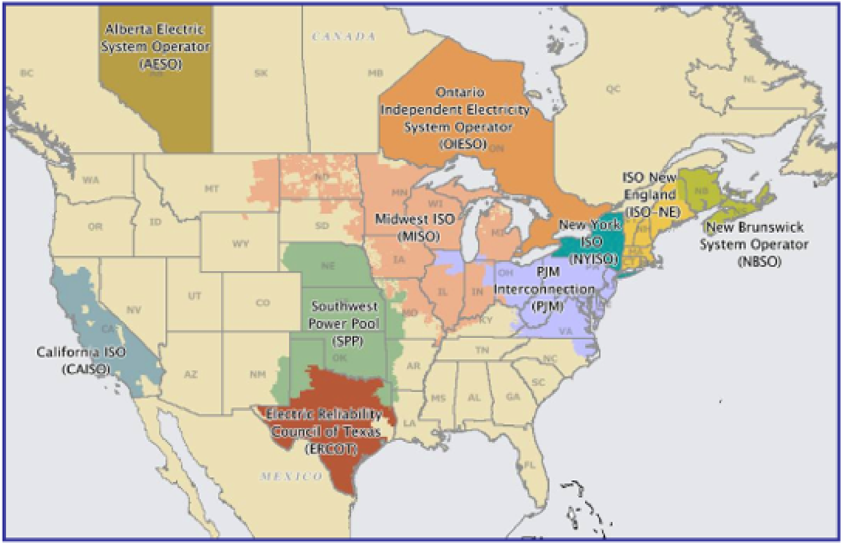
Outline

I. The Economics of Electricity Markets

Dispatching electricity markets

- Basic structure is typically designed around a wholesale market for electricity.
- Generators submit bids for electricity every day!
  - The complexity of these bids varies significantly across markets
    - Bid just one price for energy vs. include start up costs.
    - Have separate products for capacity and energy vs. only energy.
    - Etc.
- Demand also submits bids for electricity
  - Can be sloped or not
- Lots of other details that we will discuss
  - Price caps, "capacity markets", etc.

US liberali

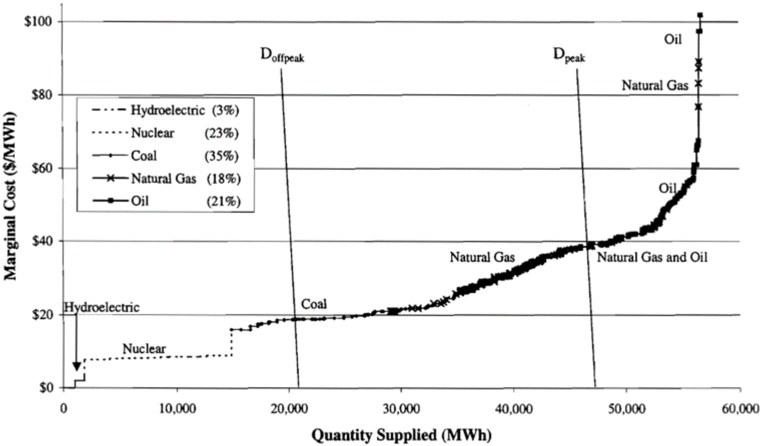

Alberta Electric System Operator (AESO)
CANADA
Ontario Independent Electricity System Operator (OIESO)
ISO New England (ISO-NE)
New Brunswick System Operator (NBSO)
New York ISO (NYISO)
Midwest ISO (MISO)
PJM Interconnection (PJM)
Southwest Power Pool (SPP)
California ISO (CAISO)
Electric Reliability Council of Texas (ERCOT)
MEXICO

An example: bidding in Chicago

- Imagine a power company in Chicago.
- It will offer its power on a *daily basis* to the PJM market.
  - The typical offer will consist of several price-quantity offers for every hour of the day.
  - Example: at 8 am, the firm is willing to produce 200 MWh as long as the price is at least \$45/MWh with one of their plants.
- Many other companies will also offer their power at the PJM market.
- The system operator will collect all the bids from all the power plants.
- It will then cross supply with demand and determine the marginal price that all accepted units get.

A supply example for PJM

Figure 2. Competitive supply and demand in Pennsylvania-New Jersey-Maryland (PJM)



What do the bids represent?

- If the market is very competitive, the bids will tend to represent the **marginal cost of a given firm**.
- If there is market power, then firms might bid above their marginal cost, to increase prices.
- For the case of hydro power, bids will tend to represent the opportunity cost of water.
  - Note: the opportunity cost of water can be quite high for markets with limited hydro availability or during scarcity conditions (droughts).
- For renewables, bids will tend to be quite low or reflect market power considerations.

What about demand?

- Demand also participates in the market, although it is typically quite inelastic.
  - Final consumers do not directly demand power: the distribution utilities or retailers do it on their behalf.
- Big industrial consumers or commercial customers might participate in the market, and avoid consuming electricity if prices are too high.
  - Much more elastic, extensive contracting that may require firms to respond in moments of high prices.
  - Some big industrial producers participate directly as generators (co-generators, direct generation).

Nodal vs. zonal markets

- The crossing of demand and supply may or may not account for bottlenecks in the electricity grid.
  - Nodal markets: Typical in the US, each node in the grid has its own price (thousands of different marginal prices every hour).
  - Zonal markets: Typical in Europe, large areas all share the same price, e.g., Spain, Portugal, four regions in Germany, etc.

- Several studies have highlighted the advantages of having more granular prices (Green, 2007; Joskow 2008; Holmberg and Lazarczyk, 2015; Graf et al., 2020).

Day-ahead vs. real-time markets

- The crossing of demand and supply may happen at different points in time.
  - Day-ahead markets: A few hours in advance, a preliminary schedule of what will happen (most commonly with a financial committment).
  - Real-time markets: A few minutes before the dispatch happens (e.g., 5 to 30 min).

- In many areas, consumers pay the day-ahead price (or a forward price that uses the day-ahead as reference).
- Therefore, a lot of focus goes into day-ahead markets, which clear the most volume.
- After the real-time market, last-minute adjustments are handled with automatic decisions (but still receive compensation ex-post).

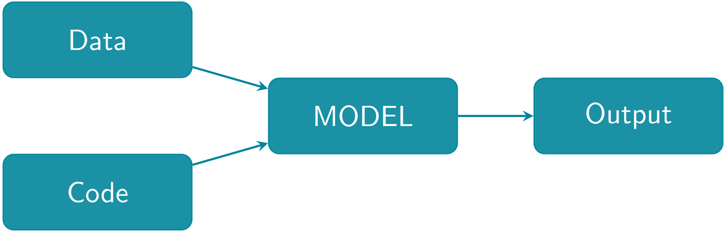
In practice, much more complex

- As we discussed, demand and supply need to balance at all time.
- Electricity markets tend to have a day-ahead auction to plan in advance.
  - Tends to clear the largest economic volume.
- But there are many follow up markets and products to ensure balance in real time.
  - Very complicated, and often market-specific!
  - Some of these markets are related to congestion.
- Electricity operators solve complex problems every hour/half-hour to determine the dispatch allocation over a wide-range of products (energy, reserves, transmission rights, etc.).

Modeling economics in electricity markets

- At its heart, all electricity market models have firms/technologies and information about demand (as a curve or fixed) to find the best allocation that ensures demand = supply (called economic dispatch).
- If the model takes into account discrete decisions about which power plants to turn on/off, it is called a unit committment problem (more difficult to solve).
- Depending on the question at hand, the electricity markets in economic analysis are modeled abstracting away from many features.
- E.g., big long-run policy questions like climate policy might be answered with a simplified version of the market.
- Depending on the question, some more detailed features need to be brought back (e.g., transmission congestion regarding renewable expansion).

Building models of electricity markets



- Model used to simulate impact of alternative configurations, profitability of investments, impacts of climate policies, etc.
- Does output for baseline match data? If not, do we need to expand code?
  - Not always, keep an eye on things that are important to our question and that we might not be matching well. A <u>model is a simplification of a complex reality.</u>

Building models of electricity markets

Common elements and options

- Supply side
  - Competitive (cost curves) or strategic (firms max profit)
  - At tech, firm, or plant level
  - With or without geography (transmission, usually with direct current approximation)
  - With or without startup costs (non-convexities)
- Demand side
  - Inelastic or responsive
  - Granular or aggregated

Horizon and temporal linkages

- Level of aggregation
  - Hourly, daily, etc.
- Links between hours
  - Every hour independent from each other vs. temporal linkages (important for storage or startup costs)
- Horizon of choice
  - Day-to-day operations
  - Seasonal water storage
  - Capacity expansion model (investment)

Outline

I. The Economics of Electricity Markets

- Overview of functioning
- Empirical analysis of electricity market performance
  - Borenstein, Bushnell, and Wolak (2002)
  - Bushnell, Mansur, and Saravia (2008)

II. Case Study: Clearing a simple CAISO market

- Overview
- Data simplification
- Modeling with JuMP

Concerns over the performance of electricity markets

- Recent high energy prices have resurfaced concerns about the performance of electricity markets:
  - Are they competitive?
  - Are they *fair*?
  - Do they have an appropriate design?
  - Is marginal pricing justified?

- A key question is to which extent firms behave as economic agents through the lens of stylized models, which can be used to benchmark competition levels.

Economics tools to analyze market performance

- Theoretical models of market design
- Empirical analysis of previous market performance
- Simulation models to examine counterfactuals (alternative market rules, configurations, input costs, etc.)

Empirical analysis of electricity markets

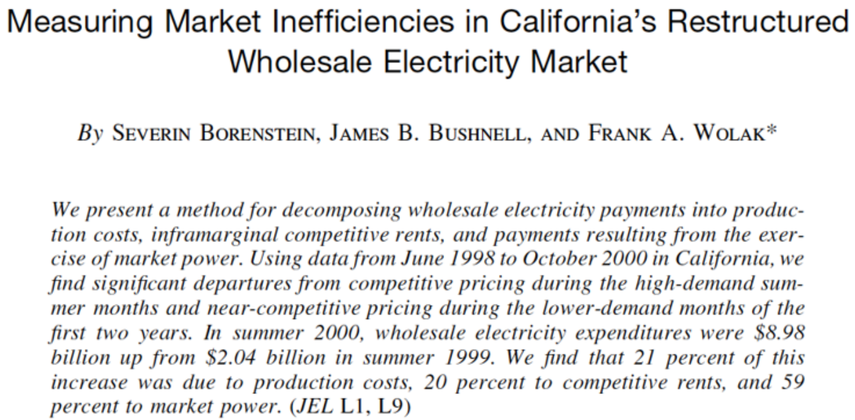
- Large literature has used electricity models to analyze the **performance of electricity markets**.
- Literature explorations:
  - How do market outcomes compare to an idealized operation of the market?
  - How do market outcomes compare to an economic model of behavior?
  - How do bidding outcomes compare to an auction model of behavior?

- I will discuss **two seminal papers** that use different approaches to modeling firm behavior (competitive vs. strategic).

Market power in electricity markets

- Market performance in deregulated wholesale markets
  - Wolfram (1999), Borenstein, Bushnell, and Wolak (2002), Wolak (2007)
- Measurements of incentives and ability to exercise market power (markup components)
  - Wolfram (1998), McRae and Wolak (2012)
- Vertical integration and market performance
  - Mansur (2007), Bushnell, Mansur, and Saravia (2008)
- Auction design in wholesale electricity markets
  - Wolak (2000, 2003) , Hortacsu and Puller (2008), Reguant (2014)
- Market power in sequential electricity markets
  - Ito and Reguant (2016)

Borenstein, Bushnell, and Wolak (2002)


Measuring Market Inefficiencies in California's Restructured Wholesale Electricity Market

By Severin Borenstein, James B. Bushnell, and Frank A. Wolak*

We present a method for decomposing wholesale electricity payments into production costs, inframarginal competitive rents, and payments resulting from the exercise of market power. Using data from June 1998 to October 2000 in California, we find significant departures from competitive pricing during the high-demand summer months and near-competitive pricing during the lower-demand months of the first two years. In summer 2000, wholesale electricity expenditures were \$8.98 billion up from \$2.04 billion in summer 1999. We find that 21 percent of this increase was due to production costs, 20 percent to competitive rents, and 59 percent to market power. (*JEL* L1, L9)

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Summary of Borenstein, Bushnell, and Wolak (2002)

- **What does the paper do?**
  1. Empirically estimate the marginal cost of production
  2. Construct a (counterfactual) competitive market price
  3. Compare it to actual market outcomes to measure market inefficiency
- **What does the paper find?**
  - Wholesale electricity expenditures in the summer of 2001 = \$8.98 billion (it was \$2.04 billion in 1999)
  - 21% of this increase was due to production costs
  - 20% to competitive rents
  - 59% to market power

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Data

- Hourly price and quantity data at Power Exchange (PX) day-ahead market from 1998-1998, settlement ISO data.
- Estimates of heat rates by power plant, O&M, pollution costs (NO_x), from the California Energy Commission.
- Spot gas prices times heat rate determines cost.
- Outages/unavailabilities from NERC.

Market Structure

TABLE 1—CALIFORNIA ISO GENERATION COMPANIES (MW)

July 1998—online capacity				
Firm	Fossil	Hydro	Nuclear	Renewable
AES	4,071	0	0	0
Duke	2,257	0	0	0
Dynegy	1,999	0	0	0
PG&E	4,004	3,878	2,160	793
Reliant	3,531	0	0	0
SCE	0	1,164	1,720	0
SDG&E	1,550	0	430	0
Other	6,617	5,620	0	4,267
July 1999—online capacity				
Firm	Fossil	Hydro	Nuclear	Renewable
AES	4,071	0	0	0
Duke	2,950	0	0	0
Dynegy	2,856	0	0	0
PG&E	580	3,878	2,160	793
Reliant	3,531	0	0	0
SCE	0	1,164	1,720	0
Mirant	3,424	0	0	0
Other	6,617	5,620	430	4,888

Source: California Energy Commission (www.energy.ca.gov).

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Methodology

1. Cost estimation
   - Based on engineering estimates
   - Need to deal with water (complicated dynamic program, simplify with "peak shaving") and "must-take" (fixed)
   - Need to estimate import supply elasticity
   - Montecarlo to control for outages, maintenance
2. Counterfactual
   - Construct marginal cost curves using above assumptions
   - Competitive equilibrium as $P = MC$.
3. Market power
   - Compare observed prices to competitive prices

Comparison to the IO literature

Similarities

- Markup calculation as the residual from marginal cost, $P = MC + \text{Markup}$

Differences

- Marginal cost not estimated, taken from engineering estimates
- Does not consider a strategic model of competition, more "non-parametric"
- Drawback: strong assumptions behind interpretation

Weighted Markups

- **Lerner Index**

$$\text{Markup} = \frac{P - MC}{P}$$

- In this setting:

$$\text{Markup} = \frac{P_{observed} - P_{competitive}}{P_{observed}}$$

- Note: Paper weights each price with quantities, more weight when total quantity is larger (after taking away "must take", which they hold fixed).

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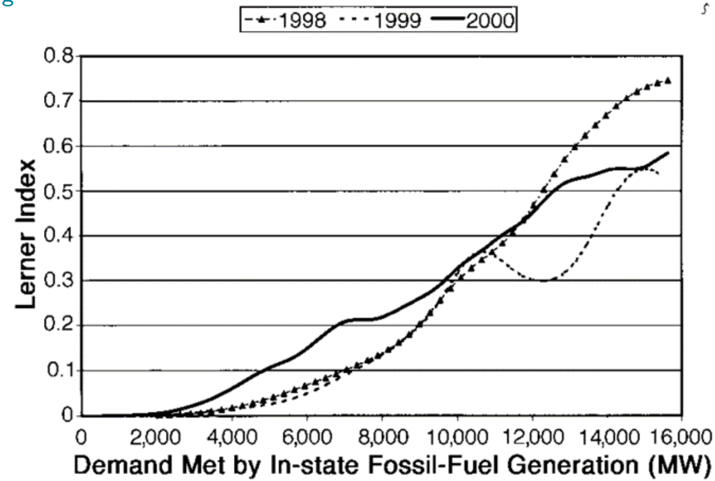
Markups increase as a function of production

Markups higher during



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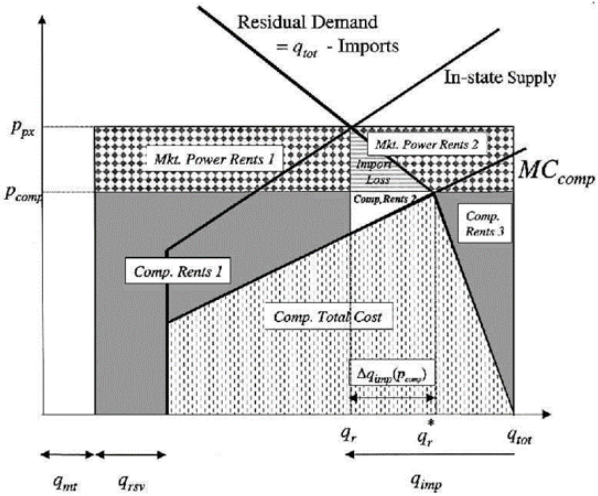
Rent division

Total wholesale market payment can be divided into the three types:

- Production costs
  - Even holding quantity fixed, potentially larger under oligopoly, specially with asymmetric firms (e.g., see Mansur 2008)
- Infra-marginal competitive rent
- Rents due to market power (higher prices)

Important to understand the difference between the three types of costs

Decomposition of expenditure



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Decomposition of expenditure

TABLE 3—PRODUCTION COSTS AND RENT DISTRIBUTION ($ MILLION) JUNE–OCTOBER

	1998	1999	2000
Total actual payments	1,672	2,041	8,977
Total competitive payments	1,247	1,659	4,529
Production costs—actual	759	1,006	2,774
Production costs—competitive	715	950	2,428
Competitive rents	532	708	2,101
Oligopoly rents	425	382	4,448
Oligopoly inefficiency—in state	31	31	126
Oligopoly inefficiency—imports	13	24	221

Bushnell, Mansur, and Saravia (2008)

Vertical Arrangements, Market Structure, and Competition: An Analysis of Restructured US Electricity Markets

By James B. Bushnell, Erin T. Mansur, and Celeste Saravia*

This paper examines vertical arrangements in electricity markets. Vertically integrated wholesalers, or those with long-term contracts, have less incentive to raise wholesale prices when retail prices are determined beforehand. For three restructured markets, we simulate prices that define bounds on static oligopoly equilibria. Our findings suggest that vertical arrangements dramatically affect estimated market outcomes. Had regulators impeded vertical arrangements (as in California), our simulations imply vastly higher prices than observed and production inefficiencies costing over 45 percent of those production costs with vertical arrangements. We conclude that horizontal market structure accurately predicts market performance only when accounting for vertical structure. (JEL L11, L13, L94)

Bushnell, Mansur, and Saravia (2008)

What does the paper do?

- Compare market performance in three US wholesale electricity markets using strategic models
  - California
  - New England
  - PJM (Pennsylvania, New Jersey, and Maryland)
- Examine which of three models fit actual market outcomes best
  - Perfect competition
  - Cournot oligopoly
  - Cournot oligopoly with vertical integration
- Analyze how the **vertical integration** of retail and wholesale parts affect the competitiveness of wholesale electricity markets

Motivation: Why California?

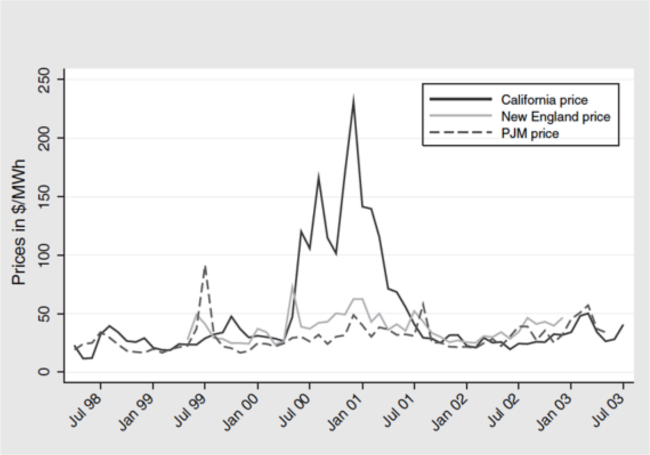

FIGURE 1. PRICE PATH IN ALL MARKETS
(California, New England, and PJM Monthly Averages)

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Comparison across the three markets

	California	New England	PJM
When did transactions start?	April, 1998	May, 1999	April, 1999
Who controls transmission lines?	California ISO (CAISO)	New England ISO (ISONE)	PJM Interconnection
Output max summer 1999 (GWh)	44.1	25.7	56.7
Load max summer 1999 (GWh)	45.9	22.3	51.7
Horizontal market concentration (HH)	620	850	1400
Import	25%	10%	little

Vertical Integration after deregulation

- **PJM**
  - Retailers retained their generation assets
  - In other words, retailers and wholesalers were vertically integrated
- **New England**
  - Divestitures of generation from vertically integrated utilities
  - However, retail utilities signed long-term supply contracts with wholesalers
  - Retailers signed contracts with the wholesaler that they previously owned
- **California**
  - No meaningful long-term contracts
  - Most electricity was sold in the pool spot market
  - Large utilities still owned some generating plants in 1999, but they were low marginal cost capacity (nuclear and hydro)

Vertical Integration and market power

Vertical integration in the three markets

- *PJM and New England:* vertically integrated or long-term contracts between retailers and wholesalers
- *California:* almost no vertical integration for high marginal cost plants

Hypothesis

- Vertically integrated firms have LESS incentives to raise wholesale prices
- This is because integrated firms make retail price commitments before committing production to their wholesale market
- On the other hand, non-integrated wholesalers have larger incentives to raise wholesale prices because they do not need to care about retail prices

Vertical arrangements in a Cournot setting

Assume profit maximizing firms

$$\pi_{i,t}(q_{i,t}, q^r_{i,t}) = p^w_t(q_{i,t}, q_{-i,t}) \cdot [q_{i,t} - q^r_{i,t}] + p^r_{i,t}(q^r_{i,t}, q^r_{-i,t}) \cdot q^r_{i,t} - C(q_{i,t})$$

Implied first order condition

$$\frac{\partial \pi_{i,t}}{\partial q_{i,t}} = p^w_t(q_{i,t}, q_{-i,t}) + [q_{i,t} - q^r_{i,t}] \cdot \frac{\partial p^w_t}{\partial q_{i,t}} - C'_{i,t}(q_{i,t}) \geq 0$$

- Key is that q^r and p^r are considered sunk at this stage.
- Firms only care about the impact of marginal price increases on the net day-ahead market quantity.
- For competitive, assume no markup term.

Data

- PJM, New England and California data.
- Similar cost data to BBW (California), Saravia (2003) for New England, and Mansur (2007) for PJM.
- Important addition with vertical arrangements and long-term contracts.
  - Vertical position inferred for vertically integrated firms
  - Publicly available data on long-term contracts for PJM and New England
  - No data for California on long-term contracts, but by construction there were limited

Results: All Hours

Variable	Mean	Median
Panel A: Peak hours (11 am to 8 pm weekdays)		
California actual	43.15	34.52
Competitive	35.01	30.88
Cournot	45.17	40.19
New England actual	55.05	33.16
Competitive	41.72	35.04
Cournot	54.63	40.44
Cournot n.v.a.	280.47	145.86
PJM actual	97.31	33.17
Competitive	35.08	33.27
Cournot	87.05	36.00
Cournot n.v.a.	1,000.00	1,000.00

- Cournot setting much better at replicating observed prices than Competitive setting
- Vertical arrangement crucial (see substantially higher prices for n.v.a rows)

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Very nice fit across all markets

Calefornia

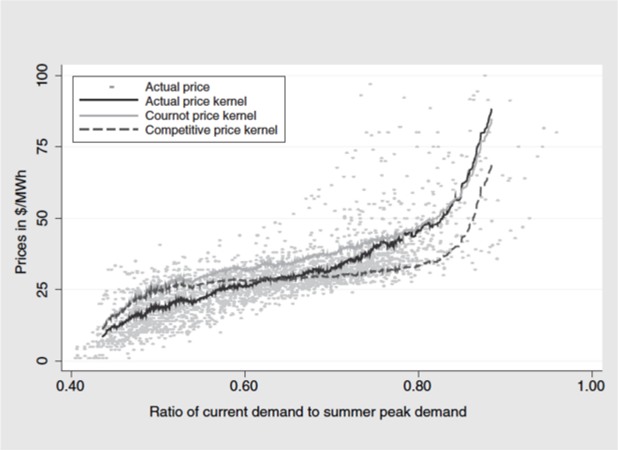

FIGURE 2. PRICES BY QUANTITY DEMANDED IN CALIFORNIA
(Actual, competitive, and Cournot price kernels)

Very nice fit across all markets

New England

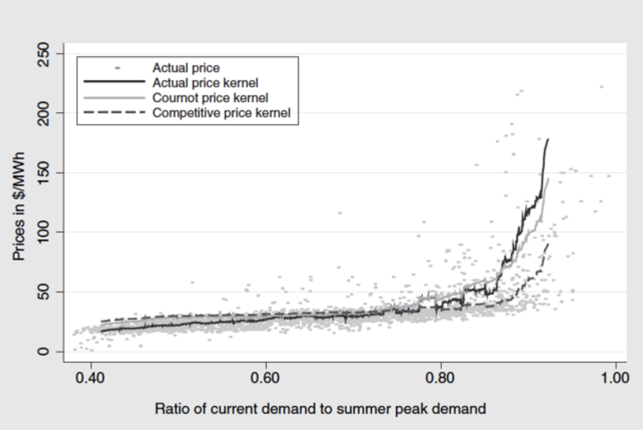

FIGURE 5. VERTICAL ARRANGEMENTS IN NEW ENGLAND
(Actual, competitive, and Cournot price kernels)

Very nice fit across all markets

PJM

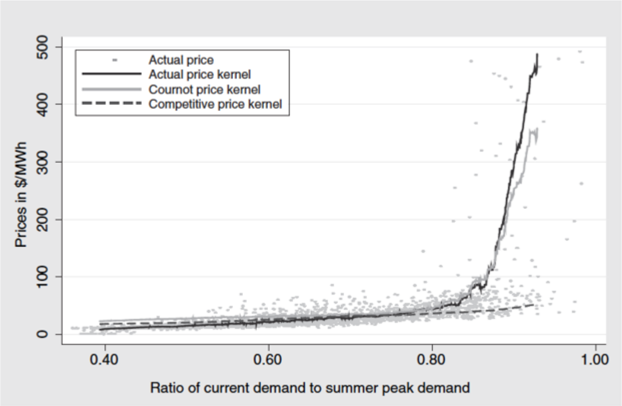

Figure 6. Vertical Arrangements in PJM
(Actual, competitive, and Cournot price kernels)

Comparison Across Hours

Variable	Mean	Median
Panel A: Peak hours (11 am to 8 pm weekdays)		
California actual	43.15	34.52
Competitive	35.01	30.88
Cournot	45.17	40.19
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PJM actual	97.31	
Competitive	35.08	
Cournot	87.05	
Cournot n.v.a.	1,000.00	
Panel B: Off-peak hours		
California actual	23.90	24.99
Competitive	26.10	27.44
Cournot	30.00	31.25
New England actual	29.18	26.61
Competitive	31.73	31.14
Cournot	32.63	30.54
Cournot n.v.a.	86.16	55.82
PJM actual	23.84	18.10
Competitive	25.42	23.78
Cournot	32.73	30.00
Cournot n.v.a.	900.57	1,000.00

Check Reguant (2014)
for a correction on markups

Potential biases due to
dynamic costs of operation

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Summary of Bushnell, Mansur, and Saravia (2008)

- Vertical arrangements are of crucial importance to explain firm behavior
- When vertical arrangements are accounted for, Cournot model gives a good fit to the data
  - Ideally, SFE. But not as tractable.
- Other work has been using the BMS framework to look at other questions.
  - E.g., Ito and Reguant (2014).

II. Case Study: Clearing a simple CAISO market

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I. The Economics of Electricity Markets

- Overview of functioning
- Empirical analysis of electricity market performance
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  - Bushnell, Mansur, and Saravia (2008)

II. Case Study: Clearing a simple CAISO market

- Overview
- Data simplification
- Modeling with JuMP

Today's application paper

- We will be building the simplest version of an electricity market with data from Reguant (2019).
- Paper has investment and retail equilibrium prices, but today we will focus on simple short-run model.
- Analogous to models in Bushnell, Mansur, and Saravia (2008) and the second stage of Ito and Reguant (2016), but without market power.
- **Main goal** is to get some familiarity about how these models are formulated as mathematical programming objects and how they are built in Julia.

Summary of Reguant (2019)

- **Question:** Examine current practice of charging renewable costs mostly to residential sector.
- **Data:** California market data to calibrate a stylized model of an electricity market with 3 types of end users (I, C, R).
- **Methods:** Ramsey pricing theory with externalities, computational tools for quant assessment.
- **Finding:** Charging residential HH cannot be justified by Ramsey pricing unless industrial sector leaks.

Motivation

- Renewable policies have grown in popularity across states in the US, and also worldwide.
- The costs and benefits from renewable policies are unevenly distributed across several margins.
  - Stakeholders.
  - Regional heterogeneity in resources (and correlation of resources with demand).
  - Heterogeneity across consumer types, e.g. residential vs commercial.
  - Heterogeneity in consumption, e.g., across income groups.

Goal: Quantify (some of) these distributional impacts under alternative policy assumptions, focusing on redistribution across sectors (customer classes). *Who to charge?*

- Carbon tax, feed-in tariff, production subsidy and renewable portfolio standards (RPS).
- We will do much more about distributional effects later in the course.

Equilibrium model for this paper

- **Model needs to solve for:**
  - Supply and demand choices, market and retail prices.
  - Investment level of each technology. This step makes model more expensive, cannot solve each hour fully separately (tomorrow).
  - Retail prices that include subsidies to renewable power, with taxes that can be split in different ways and designed optimally. This step makes the problem more expensive as well, not nice equations, need to solve iteratively many times.

Data

Use data from California to build a simulation framework, 2011-2015.

- Load data
  - Hourly, by utility and customer class (dynamic load profiles).
  - Monthly, by utility, zipcode and customer class.
- Generation data
  - Generation by type and imports, hourly.
  - Wind and solar potential based on actual production.
  - Combine with assumptions on marginal and fixed costs.

Demand

- Use hourly demand by class to account for correlation between demand and renewables.
- Make assumptions about elasticity of different sectors (residential, commercial, industrial).

Sector	Elasticity	Share
Residential	0.15	41%
Commercial	0.30	45%
Industrial	0.50	14%

Imports

- Estimate import supply from data.

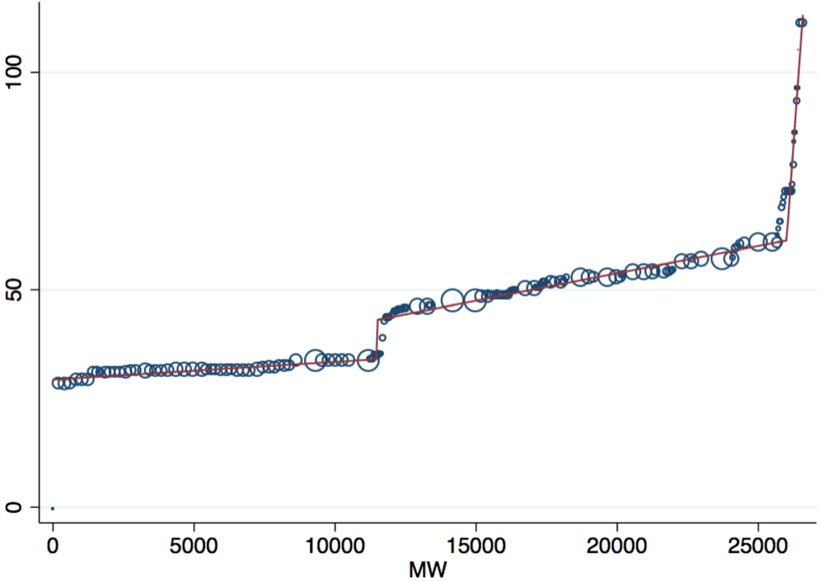
	(1) Log Imports	(2) Log Imports	(3) Log Imports
Log Price	0.3103	0.2902	0.2912
	(0.0055)	(0.0037)	(0.0032)
Observations	43,364	43,364	43,364
Weather controls	Yes	Yes	No
Year and Month FE	No	Yes	No
YearXMonth FE	No	No	Yes

Generation

- Construct incumbent supply curve based on existing generation mix for thermal plants and emissions rates.
- Take as given hourly hydro and nuclear production.
- Use EIA construction cost data from new investment to calibrate costs of new plants.

- Researchers are also starting to use k-means to simplify the number of power plants. In this application, not much machine learning was needed...

Generatio



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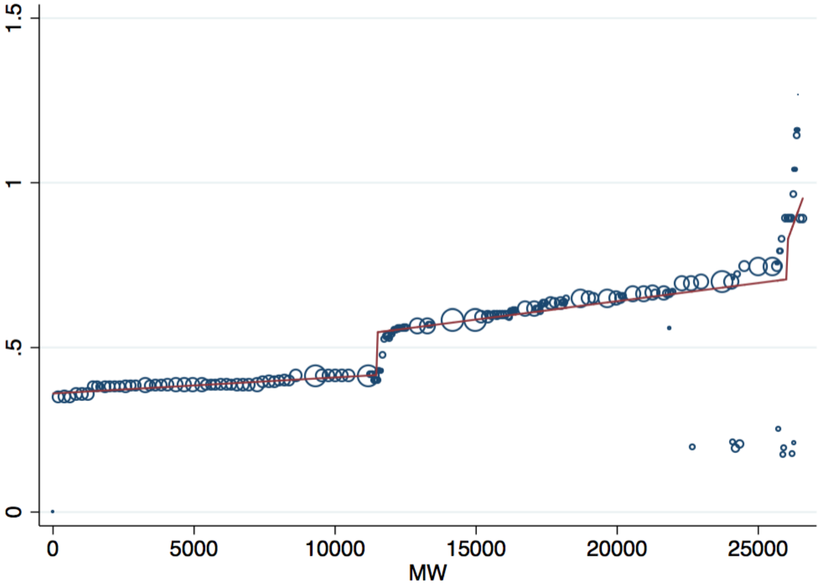
Emissions



Renewables

- Renewable output by category from 2013 onwards.
- Used to generate different renewable profiles (distribution of utilization factors during the day and over the seasons).
- Model has only a single region, so variation is limited to different technologies.
- Investment costs based also on EIA realized cost data.

Simplifying the data and the model

Simplifying the data

- You will learn how to use k-means clustering to vastly reduce the number of hours that are used.
- Results are still very similar, but it takes a much shorter time.

Simplifying the model

- The model will be simple: supply aggregated at the technology level, without hydro modeling.
- Demand will be aggregated (one category today, three in the paper).
- Model will treat firms as competitive, offering their power at their marginal cost (social planner equivalent).

Simplifying the data

- You will see more about this in the practical session.
- Key idea is to identify "representative hours" with some "weights" for how important each hour is.
- These representative hours can then be used in the model (together with the weights) to ensure that the model is representative (but runs much faster).

- *Note:* The hourly clustering is easiest, but it treats each hour as independent. Depending on the problem, clustering days or weeks might be better.
  - E.g., for a short-term battery problem, need to look at battery behavior for at least three days; for hydro, very difficult to cluster due to seasonal rains and long-term storage.

Clustering of different dimensions

- Dimension reduction techniques can be used in many ways to reduce the computational demands of electricity market models.
- Today: application simplifies the time dimension.
- Other examples:
  - Types of consumers (see later in the course, as in Cahana et al, 2022).
  - Geographical granularity to simplify nodal market data (e.g., see Mercadal, 2021; Gonzales, Ito and Reguant, 2022).
  - Types of production units to simplify technologies in the model.

The k-means clustering algorithm

- Input data: matrix where each column represents a “unit” that we want to classify, rows are the number of observations per unit.
  - Examples: what are the rows? what are the columns?
- Tuning parameter: a parameter or set of parameters to decide how much granular the clusters will be (e.g., directly chosing number of clusters n).
- Output: an assignment of units to clusters, cluster centers (representative observations) and cluster weights (how important a cluster is).

Auctions in electricity markets

- To decide supply and demand, the centralized planner clears an auction.
  - Suppliers submit willingness to produce.
  - Consumers submit willingness to pay.

- Planner maximizes the net surplus based on these offers, sometimes considering **constraints** due to the complexities of electricity generation and delivery.
- This is not an abstraction, every single day, several times, electricity market operators are solving these optimization problems.

Inputs to the auction

- At the very least:
  - Demand curve.
  - Supply curve.
- Often:
  - Some additional rules and constraints.

Our goal today

- Our goal today is to create these inputs based on the data from last week (CAISO).
- We then need to solve for the objective function.

$$\begin{aligned} \max_{q} \quad & S(q) - C(q) \\ s.t. \quad & \text{demand=supply}, \\ & \text{other constraints.} \end{aligned}$$

- We solve for the quantities that maximize the gross surplus S minus the costs of generation C.
- Implicitly or explicitly, there is a price to electricity consumption.

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Solving the model with JuMP

- JuMP makes the formulation of electricity dispatch models relatively seamless.
- One code to express the model, one can then call several solvers depending on the needs.
- I will give you a “hint” of what JuMP can do.
- Example of highly configurable electricity expansion model based on Julia + JuMP:
  - https://github.com/GenXProject/GenX

Ingredients to a mathematical model

- Parameters/Inputs
- Variables
- Constraints
- Objective function
- Sense of the objective function
- The solver we want to use

Note: In mathematical programming, the terms ‘variables‘ and ‘parameters‘ are used the opposite way as in econometrics! Variables: what we are trying to solve. Parameters: what we already have, the inputs.

Solvers

- There is an array of optimization resources that are tailored to be particularly efficient in certain problems.
- Developed/used more in engineering and operations research.
- Examples:
  - Quadratic programs
  - Linear programs with integer variables
  - Nonlinear programs with integer variables
  - Programs with complementary conditions

Next class

- **Supply II**
  - What environmental policies affect electricity markets?
  - How can we model these regulations?
  - Practicum: add investment