Day 4: Supply II - Enhancing the model

We will continue our modeling exercise by adding investment to the model.

This will allow us to discuss the rationale for how electricity markets work and to consider the policy impacts of alternative energy transition policies.

The data and code are based on the paper "The Efficiency and Sectoral Distributional Implications of Large-Scale Renewable Policies," by Mar Reguant.

We first load relevant libraries, same as last session.

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

from pyomo.environ import (
    ConcreteModel, Var, Param, Set, RangeSet, Constraint, Objective, maximize, minimize,
    NonNegativeReals, Reals, Binary, value, SolverFactory
)
```

61] ✓ 0.0s Python

Remember to set your path correctly:

```
dirpath = "/Users/marreguant/Dropbox/TEACHING/BSE/Electricity2026/day4/practicum/"
```

62] ✓ 0.0s Python

Building the model

We load the same data as last week, and also clean it up to simplify it further and create the demand and import curves.

```python
def load_and_prepare(dirpath: str):

    dfclust = pd.read_csv(f"{dirpath}/data_jaere_clustered.csv")
    tech = pd.read_csv(f"{dirpath}/data_technology.csv")

    # Re-scaling weights: multiply by 8.76 and normalize (so that we can interpret it as annual revenue in million USD)
    dfclust["weights"] = 8.76 * dfclust["weights"] / dfclust["weights"].sum()

    # One demand type (sum of components)
    dfclust["demand"] = (
        dfclust["q_residential"] + dfclust["q_commercial"] + dfclust["q_industrial"]
    )

    # Calibrate demand curve: demand = a - b * price
    elas = np.array([0.1, 0.2, 0.5, 0.3])
    dfclust["b"] = elas[0] * dfclust["demand"] / dfclust["price"]
    dfclust["b"] = dfclust["b"].mean()  # constant slope across t
    dfclust["a"] = dfclust["demand"] + dfclust["b"] * dfclust["price"]

    # Calibrate imports: imports = am + bm * price
    dfclust["bm"] = elas[3] * dfclust["imports"] / dfclust["price"]
    dfclust["am"] = dfclust["imports"] - dfclust["bm"] * dfclust["price"]

    # Set index names for tech
    tech.index = ["hydronuc","gas1","gas2","gas3","newgas","wind","solar"]

        # Annualization factor and fixed costs
    afactor = (1 - (1 / (1.05**20.0))) / 0.05
    tech["F"]  = tech["F"]  / afactor

    return dfclust, tech
```

Notice the weights, adding up to 8,760 hours (in thousands) to give the right monetary units

We also annualize the fixed cost of building power plants, here 20 years at 5%

```python
tech
```

✓ 0.0s

Python

	techname	heatrate	heatrate2	F	capLB	capUB	new	renewable	solar	thermal	e	e2	c	c2
ıydronuc	Hydro/Nuclear	10.000000	0.000000	0.000000	1.000	1.000	0	0	0	0	0.000000	0.000000	10.000000	0.000000
gas1	Existing 1	6.671990	0.092912	0.000000	7.500	11.500	0	0	0	1	0.360184	0.004886	23.351965	0.325193
gas2	Existing 2	9.794118	0.286247	0.000000	10.500	14.500	0	0	0	1	0.546134	0.011078	34.279413	1.001866
gas3	Existing 3	13.818120	20.535160	0.000000	0.578	0.578	0	0	0	1	0.816768	0.234476	48.363420	71.873060
newgas	New Gas	6.600000	0.000000	78.477250	0.000	100.000	1	0	0	1	0.350000	0.000000	23.351965	0.325193
wind	Wind	0.000000	0.000000	100.303234	0.000	100.000	1	1	0	0	0.000000	0.000000	0.000000	0.000000
solar	Solar	0.000000	0.000000	100.303234	0.000	100.000	1	1	1	0	0.000000	0.000000	0.000000	0.000000

Notice this is a slightly different tec file, we have the possibility to add new gas and there are fixed costs.

1) Adding investment solving "by hand"

This will update the code to compute profits. Here we will focus on natural gas investment but one can add any profit.

Other than that, we will be solving the model as if investment is already chosen, those not adding any additional complication to the social planner problem.

```
def clear_market_profit(data, tech, wind_gw=0.0, solar_gw=0.0, gas_gw=0.0, ng_price=3.5, solver_name="ipopt"):
    """
    Welfare max (NLP) without solving for optimal investment
    """
    T = len(data)

    # plain python lookups (no Params)
    c = {k: float(tech.at[k, "c"]) for k in TECHS}
    if "heatrate" in tech.columns:
        for k in ["gas1","gas2","gas3","newgas"]:
            if k in tech.index:
                c[k] = float(tech.at[k, "heatrate"]) * float(ng_price)

    F = {k: float(tech.at[k, "F"]) for k in TECHS} if "F" in tech.columns else {k: 0.0 for k in TECHS}
    capUB = {k: float(tech.at[k, "capUB"]) for k in ["gas1","gas2","gas3"] if "capUB" in tech.columns and k in tech.index}

    m = ConcreteModel()
    m.T = RangeSet(0, T-1)
    m.I = Set(initialize=TECHS, ordered=False)

    m.price   = Var(m.T, domain=Reals)
    m.demand  = Var(m.T, domain=Reals)
    m.imports = Var(m.T, domain=Reals)
```

....

This is the same as before, just adding gas investment, we will use the function itself to find the best investment but investment is exogenous (not solved for)

```python
if term.lower() in ("optimal", "locallyoptimal", "locally_optimal", "locally optimal"):
    price   = np.array([value(m.price[t])   for t in m.T])
    demand  = np.array([value(m.demand[t])  for t in m.T])
    imports = np.array([value(m.imports[t]) for t in m.T])
    q = {k: np.array([value(m.q[t,k]) for t in m.T]) for k in TECHS}

    w_arr = data["weights"].to_numpy()
    avg_price = float(np.sum(price * w_arr)/np.sum(w_arr))

    # cost accounting (matches Julia)
    cost = float(np.sum(w_arr * (sum(c[k] * q[k] for k in TECHS) + (imports - data["am"].to_numpy())**2 / (2.0 * data["bm"].to_nu

    # welfare/surplus (Julia subtracts fixed costs outside objective)
    surplus = float(np.sum([data.weights[t] * (value(m.gs[t]) - value(m.costs[t])) for t in m.T])
                - F.get("newgas", 0.0) * float(gas_gw)
                - F.get("wind",   0.0) * float(wind_gw)
                - F.get("solar",  0.0) * float(solar_gw))

    profit_gas = (None if gas_gw == 0 else
                float(np.sum([data.weights[t] * (price[t] - c["newgas"]) * q["newgas"][t] / float(gas_gw) for t in range(T)]) -
    profit_wind = float(np.sum([data.weights[t] * (price[t] - c["wind"]) * data.wind_cap[t] for t in range(T)]) - F.get("wind", 0
    profit_solar = float(np.sum([data.weights[t] * (price[t] - c["solar"]) * data.solar_cap[t] for t in range(T)]) - F.get("solar

    return {
        "status": term,
        "avg_price": avg_price,
        "price": price,
        "quantity": q,      # dict: tech -> (T,) array
        "imports": imports,
        "demand": demand,
        "cost": cost,
        "profit_gas": profit_gas,
        "profit_wind": profit_wind,
        "profit_solar": profit_solar,
        "surplus": surplus,
    }
```

Only difference is that I added total surplus and firm profits to the output, so that we can look for the optimal point for different values of investment.

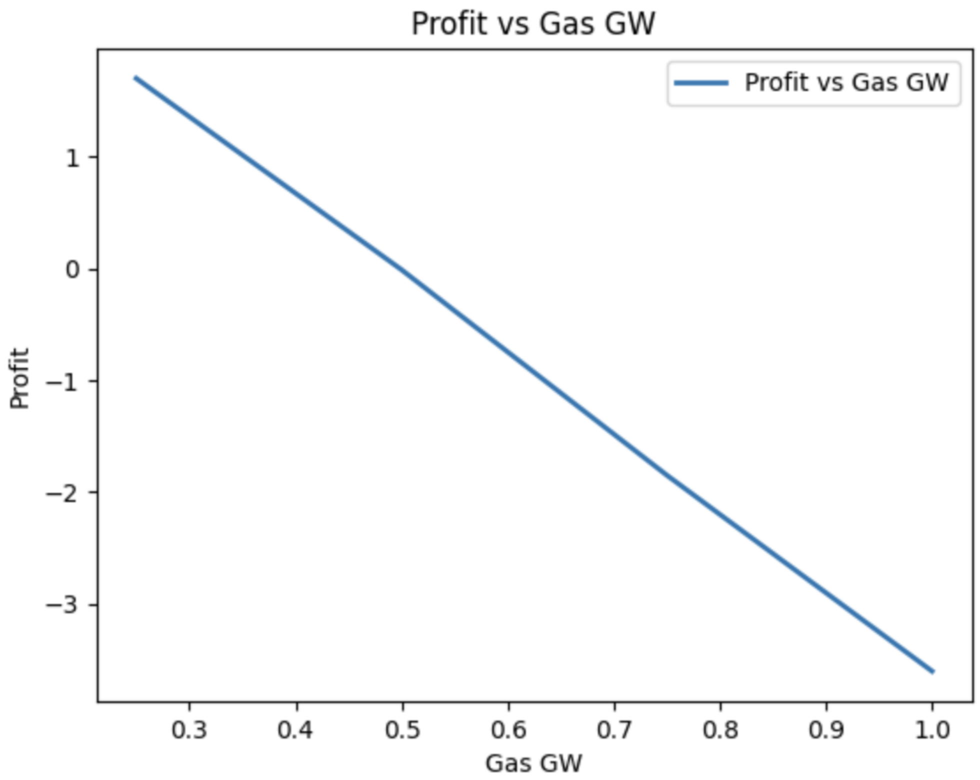
```python
gas_grid = np.arange(0.0, 1.0 + 1e-9, 0.25)   # 0.0:0.25:5.0 in Julia

profits = np.array([
    clear_market_profit(dfclust, tech, solar_gw=0.0, wind_gw=0.0, gas_gw=g)["profit_gas"]
    for g in gas_grid
], dtype=float)

plt.figure()
plt.plot(gas_grid, profits, lw=2, label="Profit vs Gas GW")
plt.xlabel("Gas GW")
plt.ylabel("Profit")
plt.title("Profit vs Gas GW")
plt.legend(loc="upper right")
plt.show()
```

✓ 1.2s

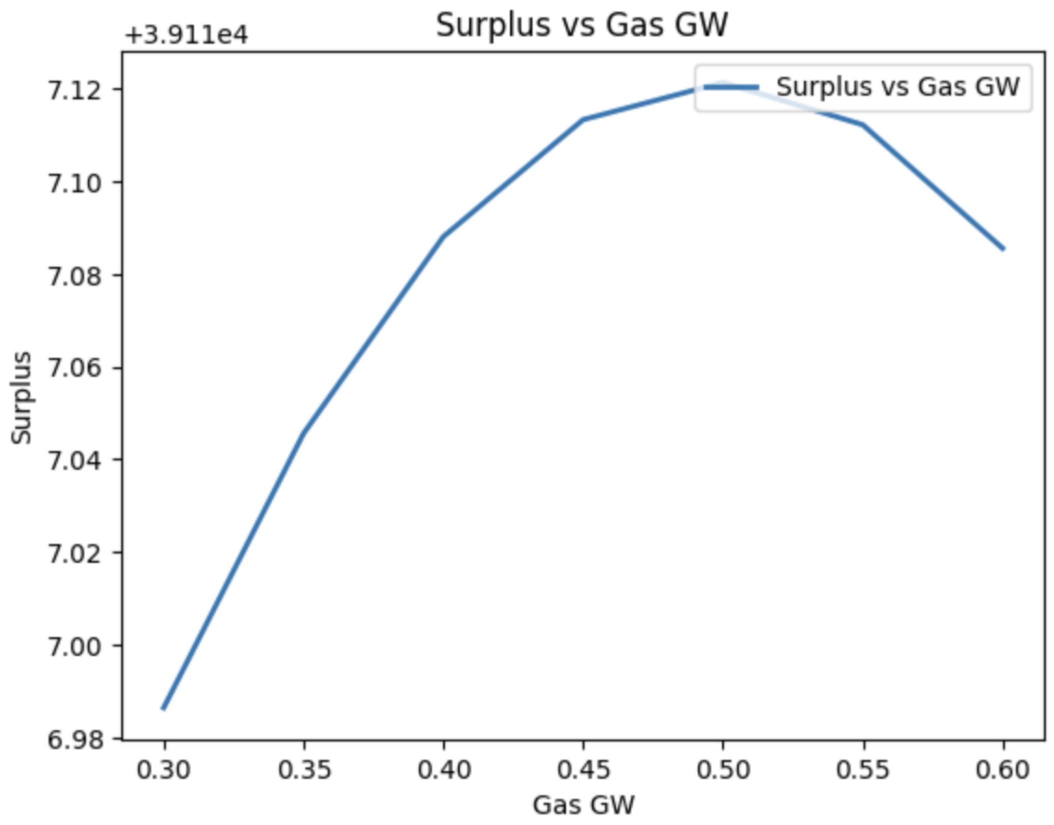
Notice a quick way to evaluate and plot an outcome for different input parameters



We can see the zero profit condition at around 0.5 GW

```python
# This is where surplus is maximized! (competitive equilibrium)
gas_grid = np.arange(0.3, 0.6 + 1e-9, 0.05)
surplus = np.array([clear_market_profit(dfclust, tech, solar_gw=0.0, wind_gw=0.0, gas_gw=g)["surplus"] for g in gas_grid], dtype=fl
plt.figure()
plt.plot(gas_grid, surplus, lw=2, label="Surplus vs Gas GW")
plt.xlabel("Gas GW")
plt.ylabel("Surplus")
plt.title("Surplus vs Gas GW")
plt.legend(loc="upper right")
plt.show()
```

✓ 1.6s Pytho



This is a competitive equilibrium. The zero profit condition maximizes surplus in the market.

This is consistent with the Boiteux result: the rents created by the prices in the market lead to optimal entry.

2) Adding investment with social planner

Here we maximize surplus directly by adding fixed costs to objective, much better as there are several technologies so the “zero profit” trick is not very elegant

We are now ready to clear the market. We will **maximize welfare** using a non-linear solver.

$$\max\ CS - Costs - FixedCosts$$
$$\text{s.t. operational constraints, market clearing.}$$

Notice that we added the fixed costs to the problem, as we will be solving for the optimal level of wind and solar investment.

```
def clear_market_invest(data, tech, ng_price=3.5, tax=50.0, Kmax=50.0, solver_name="ipopt"):
    """
    Welfare max with endogenous investment (NLP)
    """
    T = len(data)

    # plain python lookups (no Params)
    c = {k: float(tech.at[k, "c"]) for k in TECHS}
    if "heatrate" in tech.columns:
        for k in ["gas1","gas2","gas3","newgas"]:
            if k in tech.index:
                c[k] = float(tech.at[k, "heatrate"]) * float(ng_price)

    e = {k: float(tech.at[k, "e"]) for k in TECHS} if "e" in tech.columns else {k: 0.0 for k in TECHS}
    F = {k: float(tech.at[k, "F"]) for k in TECHS} if "F" in tech.columns else {k: 0.0 for k in TECHS}
    capUB = {k: float(tech.at[k, "capUB"]) for k in ["gas1","gas2","gas3"] if "capUB" in tech.columns and k in tech.index}

    m = ConcreteModel()
    m.T = RangeSet(0, T-1)
    m.I = Set(initialize=TECHS, ordered=False)
    m.J = Set(initialize=INV_TECHS, ordered=False)

    m.price   = Var(m.T, domain=Reals)
    m.demand  = Var(m.T, domain=Reals)
    m.imports = Var(m.T, domain=Reals)
```

```python
m.q     = Var(m.T, m.I, domain=NonNegativeReals)
m.costs = Var(m.T, domain=Reals)
m.gs    = Var(m.T, domain=Reals)

m.K      = Var(m.J, bounds=(0.0, float(Kmax)))  # capacity investments

# Objective: gross surplus − variable costs − fixed costs
m.obj = Objective(
    expr=sum(data.weights[t] * (m.gs[t] − m.costs[t]) for t in m.T)
        − sum(F.get(k) * m.K[k] for k in m.J),
    sense=maximize
)

# Market clearing
m.demand_curve  = Constraint(m.T, expr={t: m.demand[t]  == data.a[t]  − data.b[t]  * m.price[t] for t in m.T})
m.imports_curve = Constraint(m.T, expr={t: m.imports[t] == data.am[t] + data.bm[t] * m.price[t] for t in m.T})
m.market_clear  = Constraint(m.T, expr={t: m.demand[t]  == sum(m.q[t,k] for k in m.I) + m.imports[t] for t in m.T})

# Surplus + costs
m.surplus_def = Constraint(
    m.T,
    expr={t: m.gs[t] == (data.a[t] − m.demand[t]) * m.demand[t] / data.b[t] + (m.demand[t]**2) / (2.0 * data.b[t])
          for t in m.T}
)
m.cost_def = Constraint(
    m.T,
    expr={t: m.costs[t] == sum((c[k] + float(tax) * e[k]) * m.q[t,k] for k in m.I)
                        + (m.imports[t] − data.am[t])**2 / (2.0 * data.bm[t])
          for t in m.T}
)
```

I define costs and gross surplus as separate variables to make everything more readable

We have a new variable K, investment

The objective function now includes the cost of capital

As an update, notice that we allow for a carbon tax in the model penalizing dirty production

```
# Capacity constraints
m.cap_hydronuc = Constraint(m.T, expr={t: m.q[t,"hydronuc"] <= data.hydronuc[t] for t in m.T})
if capUB:
    m.cap_gas123 = Constraint(m.T, Set(initialize=list(capUB)),
                              expr={(t,k): m.q[t,k] <= capUB[k] for t in m.T for k in capUB})

m.cap_newgas = Constraint(m.T, expr={t: m.q[t,"newgas"] <= m.K["newgas"] for t in m.T})
m.cap_wind   = Constraint(m.T, expr={t: m.q[t,"wind"]   <= m.K["wind"]  * data.wind_cap[t]  for t in m.T})
m.cap_solar  = Constraint(m.T, expr={t: m.q[t,"solar"]  <= m.K["solar"] * data.solar_cap[t] for t in m.T})
```

Notice that now capacity for new sources is defined by the capital that we chose to install in the model (m.K)

```python
res = SolverFactory(solver_name).solve(m, tee=False)
term = str(res.solver.termination_condition)

if term.lower() in ("optimal", "locallyoptimal", "locally_optimal", "locally optimal"):
    price   = np.array([value(m.price[t])   for t in m.T])
    demand  = np.array([value(m.demand[t])  for t in m.T])
    imports = np.array([value(m.imports[t]) for t in m.T])
    q = {k: np.array([value(m.q[t,k]) for t in m.T]) for k in TECHS}
    K = {k: float(value(m.K[k]))           for k in INV_TECHS}

    w_arr = data["weights"].to_numpy()
    avg_price = float(np.sum(price * w_arr) / np.sum(w_arr))

    cost = np.array([value(m.costs[t]) for t in m.T])

    profit_gas = float(np.sum([data.weights[t] * (price[t] - c["newgas"]) * q["newgas"][t] for t in range(T)])
                    - F.get("newgas", 0.0) * K["newgas"])

    surplus = float(np.sum([data.weights[t] * (value(m.gs[t]) - value(m.costs[t])) for t in m.T])
                 - F.get("newgas", 0.0) * K["newgas"]
                 - F.get("wind",   0.0) * K["wind"]
                 - F.get("solar",  0.0) * K["solar"])

    return {
        "status": term,
        "avg_price": avg_price,
        "price": price,
        "quantity": q,   # dict: tech -> (T,) array
        "imports": imports,
        "demand": demand,
        "cost": cost,    # (T,) array (like Julia costs[t])
        "K": K,
        "profit_gas": profit_gas,
        "surplus": surplus,
    }
```

We solve and output as before

```
results = clear_market_invest(dfclust, tech, ng_price=3.5, tax=0.0)
print("Investment SP:", results["status"], "avg_price:", results.get("avg_price"), "newgas:", results.get("K"))
```

✓ 0.3s

```
Investment SP: optimal avg_price: 31.96443239670289 newgas: {'newgas': 0.4983285298502584, 'wind': -8.524553279176704e-09
```

We get the same equilibrium as before

What if we add a carbon tax?

```
results = clear_market_invest(dfclust, tech, ng_price=3.5, tax=50.0)
print("Investment SP:", results["status"], "avg_price:", results.get("avg_price"), "K:", results.get("K"))
```

✓ 0.1s

```
Investment SP: optimal avg_price: 39.35908765296707 K: {'newgas': -9.972562410791536e-09, 'wind': 26.05734503353563, 'solar': -9.965
```

Only wind now

We modify our mixed integer code with an additional "dummy" variable that allows for a corner solution at zero investment. If at zero investment firms make no money, then the zero profit condition is negative. Otherwise, the zero profit condition must be zero.

```
def clear_market_foc(data, tech, ng_price=3.5, tax=50.0, M=1e4, Kmax=50.0, solver_name="highs"):
    """
    FOC-based equilibrium with investment (MILP), JuMP -> Pyomo.
    """
    T = len(data)

    # plain python lookups (no Params)
    c = {k: float(tech.at[k, "c"]) for k in TECHS}
    if "heatrate" in tech.columns:
        for k in ["gas1","gas2","gas3","newgas"]:
            if k in tech.index:
                c[k] = float(tech.at[k, "heatrate"]) * float(ng_price)

    e = {k: float(tech.at[k, "e"]) for k in TECHS} if "e" in tech.columns else {k: 0.0 for k in TECHS}
    F = {k: float(tech.at[k, "F"]) for k in TECHS} if "F" in tech.columns else {k: 0.0 for k in TECHS}
    capUB = {k: float(tech.at[k, "capUB"]) for k in ["gas1","gas2","gas3"] if "capUB" in tech.columns and k in tech.index}

    m = ConcreteModel()
    m.T = RangeSet(0, T-1)
    m.I = Set(initialize=TECHS, ordered=False)
    m.J = Set(initialize=INV_TECHS, ordered=False)

    # vars
    m.price   = Var(m.T, domain=Reals)
    m.demand  = Var(m.T, domain=Reals)
    m.imports = Var(m.T, domain=Reals)

    m.q      = Var(m.T, m.I, domain=NonNegativeReals)
    m.shadow = Var(m.T, m.I, domain=NonNegativeReals)
```

We can repeat optimal entry with mixed integer programming, allowing for conduct/market power and investment.

```
m.u1 = Var(m.T, m.I, domain=Binary)  # used
m.u2 = Var(m.T, m.I, domain=Binary)  # at cap
m.u3 = Var(m.J, domain=Binary)       # built
```

We have a new binary variable for building a technology

```
m.lb_newgas = Constraint(m.T, expr={t: m.q[t,"newgas"] >= m.K["newgas"] - float(M) * (1 - m.u2[t,"newgas"]) for t in m.T})
m.lb_wind   = Constraint(m.T, expr={t: m.q[t,"wind"]   >= m.K["wind"]  * data.wind_cap[t]  - float(M) * (1 - m.u2[t,"wind"]
m.lb_solar  = Constraint(m.T, expr={t: m.q[t,"solar"]  >= m.K["solar"] * data.solar_cap[t] - float(M) * (1 - m.u2[t,"solar"
```

The u2 condition works a bit different here but achieves the same outcome. Key: we avoid multiplying two variables, u2 and K.

```
# profits (only for investable techs)
m.profit_newgas = Constraint(expr=m.profit["newgas"] == sum(data.weights[t] * m.shadow[t,"newgas"] for t in m.T) - F.get("newgas'
m.profit_wind   = Constraint(expr=m.profit["wind"]   == sum(data.weights[t] * m.shadow[t,"wind"]  * data.wind_cap[t]  for t in m
m.profit_solar  = Constraint(expr=m.profit["solar"]  == sum(data.weights[t] * m.shadow[t,"solar"] * data.solar_cap[t] for t in m

# investment constraints (zero-profit logic)
m.zp_ub = Constraint(m.J, expr={k: m.profit[k] <= 0.0 for k in m.J})
m.zp_lb = Constraint(m.J, expr={k: m.profit[k] >= -float(M) * (1 - m.u3[k]) for k in m.J})
m.K_link = Constraint(m.J, expr={k: m.K[k] <= float(M) * m.u3[k] for k in m.J})

res = SolverFactory(solver_name).solve(m, tee=False)
term = str(res.solver.termination_condition)
```

We define profit and establish that, if there is any investment, profits need to be zero.

```python
invest_foc = clear_market_foc(dfclust, tech, ng_price=3.5, tax=0.0)
print("Investment FOC:", invest_foc["status"], "avg_price:", invest_foc.get("avg_price"), "K:", invest_foc.get("K"))
```

✓ 0.3s Python

```
Investment FOC: optimal avg_price: 280.0084283041977 K: {'newgas': 0.4983286123096766, 'wind': 0.0, 'solar': 0.0}
```

```python
invest_foc_tax = clear_market_foc(dfclust, tech, ng_price=3.5, tax=50.0)
print("Investment FOC w/ tax:", invest_foc_tax["status"], "avg_price:", invest_foc_tax.get("avg_price"), "K:", invest_foc_tax.get("K
```

✓ 1.6s Python

```
Investment FOC w/ tax: optimal avg_price: 344.7856084860571 K: {'newgas': 0.0, 'wind': 26.057345177402958, 'solar': 0.0}
```

We recover the original equilibrium and the equilibrium with the tax.

Exercise 1

Add carbon taxes to the model. Note that emissions are equal to the emissions rate times the quantity. The cost should be equal to the tax. (in class)

Exercise 2

Add a subsidy to the model. One can think of the subsidy as a negative cost to renewable power. However, it is important that you also include a penalty to the overall subsidy spending of you want to get at the overall welfare. Important however to solve the model first without worrying about where the subsidy money comes from. Otherwise, the planner will see that a subsidy is distorting outcomes.

Follow-up exercises

1. Consider a tax and a subsidy that reach the same target of emissions. What are the costs of each policy? How are the different technologies and components of welfare affected? [Note: This will require you to include emissions as an input or an output to the function.]