# A Model of Search with Price Discrimination[*]

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**Abstract**

We introduce observable heterogeneity across buyers into a model of simultaneous search. Buyers' differences are informative about their *willingness to search*, giving rise to price discrimination even if they all have the same *willingness to pay*. We analyze and compare equilibrium outcomes when price discrimination is allowed and when it is not. We find that the price comparison across consumers as well as the effects of banning price discrimination critically depend on the elasticity of the search cost distribution. Interestingly, for normally distributed search costs, there is an inverted U-shape relationship between prices and buyers' size. Similarly, a ban on price discrimination hurts small and large buyers, to the benefit of the medium-size ones.

**Keywords:** third-degree price discrimination, search, bid solicitation, competition.
**JEL Codes:** D43, D83.

## 1 Introduction

Firms often engage in third-degree price discrimination, i.e., they provide the same good or service at various prices, depending on the consumers' observable characteristics. This is the case whenever such characteristics provide information on the consumers' *willingness to pay*, their *willingness to switch*, their *willingness to bargain*, or their *willingness to search.* While the existing literature has extensively analyzed the first three sources of price discrimination, to date there has been little work regarding the impact of differences in buyers' *willingness to search.* In this paper we focus on this issue by analyzing the impact of search frictions on price discrimination in imperfectly competitive markets.

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We propose a model in which heterogeneous buyers engage in costly search and sellers compete to serve them. Buyers differ in their search costs as well as in their size,[1] both of which determine their incentives to engage in search (extensive margin) as well as to search actively (intensive margin). Sellers observe the buyers' size, but do not observe their realized search costs, or actual search behavior. However, sellers form beliefs about it based on the buyers' size. Since sellers compete more vigorously when they believe that buyers have searched more, a key question whether sellers expect this to be the case for either larger or smaller buyers. We show that the answer to this question critically depends on the shape of the search cost distribution.[2]

To fix ideas, one can think of households searching for private health insurance and schooling (families differ in size), for mortgages and moving services (for bigger or smaller houses), or for utility services such as water, electricity or gas (for higher or lower consumption levels). In these cases, when quoting prices, service providers observe the household's size, from which they can infer relevant information regarding their willingness to search. In particular, bigger households stand to gain more from search given that they benefit more from per-unit price discounts ("gain-from-search effect").[3] However, a household's size is not a perfect signal of their actual search (e.g. how many quotes they have requested from insurers, schools, banks, movers or utility companies), as search intensity also depends on their private search costs. However, their decision to participate in search is also informative about their search costs. Indeed, those households who actually engage in search signal that they have, all else equal, lower search costs than those who do not search ("signalling-through-participation effect"). Precisely because larger households have more to gain from search, their search costs are expected to be larger, conditional on search. Hence, the size of a household who is actively searching for price quotes conveys countervailing information regarding its actual search intensity. If the "gain-from-search effect" dominates, sellers will compete strongly for larger households, who will then obtain better deals. The opposite is true if the "signalling-through-participation effect" dominates instead.

The above trade-off can be resolved by analyzing the elasticity of the search cost distribution: if it is decreasing (increasing), bigger (smaller) buyers are expected to search more, which in turn leads sellers to compete more aggressively to serve them.[4] Several commonly-used distributions, such as the exponential or the Pareto distribution, depict monotonically decreasing elasticities,

[1] Similar results would arise if we introduced other sort of observable differences across buyers: e.g., differences in valuations, or differences in the search cost distribution functions.

[2] See Hortaçsu and Syverson (2004), Hong and Shum (2006), Moraga and Wildenbeest (2008) and De los Santos et al. (2012) for papers that perform this task in various contexts.

[3] Another common feature is that in all these cases, buyers find it difficult to misreport their true type (in this case, their size). For instance, families cannot pretend to have fewer or more kids than the ones they actually have, they cannot misreport the value of their houses in a mortgage as these are typically audited, they cannot pretend to be moving a different number of items than the ones they actually have, and cannot lie on the amount of utility services they actually consumed.

[4] Intuitively, for a household to be willing to engage in search (extensive margin), its search cost must be below a high threshold; to be willing to search more than once (intensive margin), its search cost must be below a lower threshold. Hence, when the search cost distribution is more elastic at the upper (lower) deciles of the search cost distribution, an increase in the buyer's size affects more her likelihood to engage in search than her search intensity. For iso-elastic functions (e.g. when search costs are uniformly distributed), the two effects cancel out leading to no price discrimination in equilibrium.

thus implying that larger buyers indeed pay lower prices. However, other distributions have non-monotonic elasticities (e.g. the Normal or the Gamma distributions), thus leading to prices that are not always decreasing in size.[5] Interestingly, with normally distributed search costs, the small and the large consumers pay the lowest prices. The medium-sized consumers are not small enough to benefit from signaling a low search cost when they engage in search, and they are not large enough to benefit from high gains from search.

The elasticity of the search cost distribution is also a key determinant of the distributional implications of banning price discrimination. To shed light on this issue, we characterize equilibrium pricing when firms are forced to set uniform prices. We show that firms charge the same prices to all buyers, just as if they were facing the average consumer. Hence, differences in the prices paid by the various consumers simply reflect differences in their search intensities. For the same logic as before, the elasticity of the search cost distribution determines whether larger or smaller buyers solicit more quotes, thus giving rise to expected prices (conditional on search) that are either decreasing or increasing in consumers' size. For instance, with decreasing elasticities, small buyers solicit fewer quotes than the large buyers (conditional on search). Hence, on average small buyers may end up paying higher prices while participating in search less often as compared to the large ones.

Allowing or banning price discrimination changes buyers' incentives to search and, as a consequence, may either weaken or strengthen the effects of search costs on competition, with distributive effects across consumers. Again, the shape of the search cost distribution is a key determinant of such effects. For instance, if the search cost distribution has a decreasing elasticity, smaller buyers face lower prices, search less (conditionally on search), and engage in search more often when a ban on price discrimination is in place. Hence, uniform pricing makes small consumers better off and large consumers worse off relative to price discrimination. However, the opposite results hold true under search cost distributions with increasing elasticities.

In sum, our results highlight the relevance of the elasticity of the search cost distribution in determining the pricing patterns and the distributive effects of banning price discrimination across consumers. Ultimately, this suggests that the desirability of price discrimination has to be assessed on an industry-by-industry basis.

**Related literature** To the best of our knowledge, all existing papers in the search literature assume *ex-ante* identical consumers, i.e., they all have unit demands and equal valuations of the good, and either their search costs are also equal, or sellers do not observe them. Hence, these papers are not well equipped to analyze the interaction between search costs and price discrimination. It is well known that search costs generate equilibrium price dispersion (including Varian, 1980; Burdett and Judd, 1983; Stahl, 1989; Janssen and Moraga, 2004, among others), which in turn implies that consumers who search more end up paying lower prices even when charged equal prices. However, such price differences due to price dispersion are not to be confounded with the price differences

[5] For instance, in the case of online books, Hong and Shum (2006) and De los Santos et al. (2012) find that the distribution of search costs depicts non-monotonic elasticities.

that arise due to price discrimination *per se*, i.e., when sellers charge different prices to different buyers.

Our paper also departs from most of the existing search literature in that we allow buyers to endogenously choose whether to engage in search or not. Endogenous participation is a key driver of price discrimination because the decision to participate in search is informative about the buyer's search cost.[6] A notable exception is Moraga et al. (2017), who develop a sequential search model with differentiated products and endogenous search. They show that a reduction in search costs can give rise to price increases as changes in the pool of consumers who engage in search make the market demand more inelastic. Our analysis shares some of the driving forces in Moraga et al. (2017), but relies on different assumptions and pursues different objectives. In particular, in Moraga et al. (2017) firms have no ex-ante information about individual buyers' characteristics, leaving no scope for price discrimination.

The recent literature on bidder solicitation also endogenizes search decisions. In particular, Lauermann and Wolinsky (2017) allows buyers to choose how many prices to solicit from a set of potential sellers at a *solicitation cost*, similar to the *search cost* in our paper (see also Lauermann and Wolinsky, 2016). The two analyses differ in one key aspect: in Lauermann and Wolinsky (2017), the buyer is privately informed about the value of the good (common-value setting); in our paper, she is privately informed about the solicitation or search cost (private-value setting). Despite these differences, both setups share an important ingredient: the buyer's participation decision (in solicitation or in search) provides a signal about the buyer's private information. In Lauermann and Wolinsky (2017), sellers are more likely to be solicited when the value of the good is low; hence, the solicitation effect softens competition. In our model, the signaling-through-participation effect enhances competition more for small than for large buyers as the decision to engage in search by a small buyer is more informative about her search costs being low.

A big part of the price discrimination literature restricts attention to the monopoly case (see the seminal works by Robinson, 1933 and Varian, 1985). Whereas the effects of price discrimination on total welfare and consumer surplus are generally thought to be ambiguous, Aguirre et al. (2010) and Cowan (2012) find conditions on the demand primitives for price discrimination to increase welfare and consumer surplus, respectively. More recently, attention has turned to analyzing the impacts of price discrimination in competitive frameworks (Holmes, 1989; Corts, 1998; Armstrong and Vickers, 2001). As shown by Holmes (1989), third-degree price discrimination always hurts some consumers, at least when all firms agree over which consumers are strong or weak.

The novelty of our approach is to show that price differences need not be driven by differences in valuations but rather by differences in their *willingness to search*. Indeed, in the absence of search frictions, our model would give rise to no price discrimination as all consumers have equal valuations.[7] Another important distinction between our paper and the standard price discrimina-

[6]This driver is absent in the majority of the search models as these assume that buyers search at least once (either because search costs are sufficiently low, or because the first quote is for free).

[7]In settings in which buyers differ in their valuations, the main message from standard price discrimination models (without search) is that a ban on price discrimination tends to hurt low valuation consumers to the benefit of high

tion literature is that, with some exceptions, the latter assumes that all markets are served under uniform pricing. This is unlike our paper, in which we allow for endogenous buyers' participation. As argued above, this is a key element of our model without which larger buyers would always face lower prices, regardless of the search cost distribution.

Only recently have researchers started to analyze the interaction between search and third-degree price discrimination. In a model of sequential search, Janssen and Reshidi (2018) analyze price discrimination by wholesale firms (yet, at the downstream market, firms charge equal prices to all customers). Wholesale price discrimination creates price dispersion in the downstream market, which in turn induces more search and stronger retail market competition, to the benefit of wholesalers (see also Marshall, 2020). Armstrong and Vickers (2019) allow for discrimination based on consumers' exogenous and observable search types (who can be either shoppers or non-shoppers).

Likewise, an increasing number of empirical papers have explored the link between the incentives to search and to price discriminate. Sorensen (2000) relates heterogeneity in the net benefits of searching to price discrimination in the drug market. In an analysis of the Canadian mortgage industry, Allen et al. (2014) find that changes in competition had heterogeneous effects on consumers, depending on their search costs. Byrne et al. (2019) provide evidence from a field experiment showing that electricity retailers engage in price discrimination based on consumers' ex-ante perceived search cost differences.[8] Our paper provides a model that can account for some of the empirical phenomena reported in these papers.

The remainder of the paper is structured as follows. In Section 2, we construct the model, which we solve in Section 3. In Section 4, we characterize the equilibrium when price discrimination is banned, while in Section 5 we compare the results under price discrimination and uniform prices. Section 6 of the paper concludes. All proofs are included in the Appendix.

## 2 Model Description

Consider a market in which a mass of consumers interacts with two firms selling homogeneous products. Each consumer is characterized by the number of units $q$ she is willing to buy. The distribution of buyers types in the market is captured by the continuous function $H(q)$, with $q \in \mathbb{R}^+$. Each buyer's per-unit valuation of consuming the good is normalized to 1. Buyers decide whether to *search* to find out the prices of the competing sellers, or not.[9] If a buyer does not search, her reservation utility is normalized to zero. The marginal cost of producing the good is

valuation consumers. If we had heterogeneity in valuations rather than size, this result would also hold in our model with search frictions but only as long as the search cost distribution depicted an increasing elasticity.

[8] Other empirical papers report price discrimination based on search frictions in the auto-car industry (Braido and Ledo, 2018), in wholesale food markets (Marshall, 2020) and in the trade waste market (Salz, 2017).

[9] Alternatively, one could assume that a buyer who does not search can buy the good at a default price normalized to 1. This would apply to several settings, e.g. in energy markets, consumers typically have access to energy at a regulated price, and consumers also have the option to search for competing retailers. The results of the model would remain the same.

also normalized to zero.[10]

Each buyer draws a search cost $c$ to observe the price of each seller.[11] Search costs are each buyer's private information, but it is common knowledge that $c$ is drawn from the cumulative distribution function $G(c)$, with density $g(c) > 0$, in the interval $[\underline{c}, \overline{c})$. We assume $\underline{c} \leq 0$ and $\overline{c} \geq q$ to focus on the interesting case in which some buyer types engage in search with probability one, while others never engage in search.

The timing of the game is as follows. First, each buyer observes her realized search cost and takes her search decisions: not to search, search once (picking one of the two sellers at random), or search twice. Sellers observe the size of each buyer, $q$, but do not observe her search cost nor her search behavior. Second, each seller chooses a price $b$ and each buyer buys from the seller that offered her the lowest price among the ones she observes (ties are broken randomly).[12]

We examine (symmetric) equilibria in which the buyer maximizes her utility given her correct beliefs about the sellers' pricing behavior, and the sellers maximize their profits given their correct beliefs about the buyer's search behavior.

We solve two variants of the game. In Section 3, we consider the case in which sellers can condition their pricing strategies on each buyer's size and are thus allowed to offer different prices across consumers. Instead, in Section 4, we consider the case in which sellers are forced to charge uniform prices to all consumer types.

# 3 Price Discrimination

In this section we assume that sellers are allowed to charge different prices to consumers as a function of their size. Hence, without loss of generality, we characterize the equilibrium when sellers face a single buyer of size $q$.

## 3.1 Pricing decisions

We start by analyzing sellers' pricing behavior given their expectations about the buyer's search behavior. Consider a seller's pricing decision. The seller does not observe the buyer's search behavior but believes that, *conditionally on search*, the buyer has searched once with probability $\rho$. In equilibrium, this expectation must turn out to be correct. To the extent that the buyer's size $q$ conveys information about her search behavior, sellers might hold different beliefs about $\rho$ depending on $q$. In this section, we take $\rho$ as a parameter, and we will endogenize it later in section 3.3.

[10] Both normalizations are *w.l.o.g* as long as valuations and marginal costs are assumed constant. We assume they are both constant so as not to bias the relationship between buyers' size and equilibrium outcomes through the assumptions on the shape of the utility or cost functions.

[11] We could introduce scale economies (or diseconomies) in search by assuming that the second quote costs $\delta c$, with $\delta < 1$ ($\delta > 1$). Our main results would remain qualitatively unchanged.

[12] Since sellers can condition their price offers to the size of the buyer, we are implicitly assuming that sellers engage in third-degree price discrimination and buyers cannot mis-report their size.

Our first result shows that in equilibrium, conditional on search, some buyers search once while others search twice. Hence, each seller does not know with certainty whether the buyer will not observe his price, whether he will be a monopolist over the buyer, or whether he will be competing with the other seller. The only information that sellers can infer is that the buyer's search cost must be low enough for her to be willing to search at least once.

**Lemma 1** *In a SPNE, if the buyer engages in search, the probability that she searches once is* $\rho \in (0, 1)$. *As a consequence, there does not exist pure-strategy equilibria in prices.*

The intuition for this result is well known (Burdett and Judd, 1983). On the one hand, if the buyer searches once with certainty ($\rho = 1$), the seller charges the monopoly price. However, as this would leave no surplus for the buyer, she would not search in the first place (Diamond's paradox). On the other hand, if the buyer searches twice with certainty ($\rho = 0$), sellers engage in Bertrand competition. Since all sellers would then price at marginal costs, the buyer would have incentives to search only once. As a consequence, neither the monopoly price nor the competitive price can be sustained in a SPNE.

More generally, $\rho \in (0, 1)$ rules out the existence of pure strategy equilibria regardless of the buyer's valuation:[13] starting from any arbitrary price pair, sellers would like to undercut each other until prices are so low that it becomes optimal for a seller to price at the consumer's maximum willingness to pay for the good. However, if one seller is pricing at that level, it becomes profitable for the other seller to slightly undercut it. Therefore, the equilibrium has to be in mixed strategies and, using the same Bertrand argument, it must be atomless.[14]

To distinguish the price offered by the seller from the price actually chosen by the buyer, we refer to the former as a *quote*. Let sellers use the (symmetric) quote distribution $F(b)$. Conditional on the buyer's decision to search, a seller's expected profits from pricing at $b$ are given by

$$\pi(b) = bq\left[\rho/2 + (1-\rho)(1 - F(b))\right].$$

A seller's mixed strategy strikes a balance between two opposing forces. On the one hand, sellers benefit from charging a high price to a buyer who has only searched once. This event occurs with probability $\rho/2$ (recall that both sellers are equally likely to be picked by the buyer).[15] On the other hand, sellers also benefit from charging a low price, as it is thus more likely to be chosen by a buyer who has searched twice. This event occurs with probability $(1-\rho)(1-F(b))$. Proposition 1 below characterizes the equilibrium quote distribution.[16]

[13] This result is reminiscent of Janssen and Rasmusen (2002) in which with an exogenously given probability, rival firms may be inactive. In contrast, in this paper we endogenize this probability through the analysis of consumers' search (see next section).

[14] This is in contrast to Lauermann and Wolinsky (2016), in which the common value assumption gives rise to an atom in bidders' strategies. This atom implies a failure of competition to aggregate information even when search costs are very low and competition is strong.

[15] Note that profits are represented conditional on a buyer searching. Results are unaffected if we instead compute expected profits conditional on the firm having received a quote request, i.e., if we re-scale profits by $1/(1-\rho/2)$.

[16] Hong and Shum (2006) and Moraga and Wildenbeest (2008) derive the same distribution for the case with $N > 2$ firms, but treat the $\rho$ parameters as exogenously given. Also, they only consider consumers with unit demands.

**Proposition 1** *Assume $\rho \in (0,1)$. There is a unique symmetric equilibrium quote distribution. It is atomless, and it is given by*

$$F(b) = 1 - \frac{1}{2}\frac{\rho}{1-\rho}\frac{1-b}{b}$$

*with compact support $b \in \left[\frac{\rho}{2-\rho}, 1\right]$.*

Interestingly, the buyer's size does not enter directly into the sellers' pricing strategy. However, this does not imply that sellers choose the same prices regardless of the buyer's size. As we will see in the next section, the effect of $q$ on price quotes is channeled through $\rho$, i.e., the sellers' expectation of the buyer's search strategy, which depends on the buyer's size. Indeed, an increase in $\rho$ shifts the whole quote distribution to the right in a FOSD sense. Intuitively, an increase in $\rho$ implies that sellers price less aggressively, leading to higher expected quotes. In other words, a buyer who is expected to search more (conditional on search) observes lower quotes, regardless of her actual search intensity (which also depends on her realized search cost).

Clearly, sellers never quote prices above the buyer's maximum willingness to pay. Therefore, conditionally on search, the buyer always buys from one of the sellers. In particular, if a buyer observes only one quote (which conditionally on search, occurs with probability $\rho$), she simply pays that quote, which in expectation equals

$$E[b] = \int_{\underline{b}}^{1} b dF(b).$$

However, if she observes two quotes (with probability $1-\rho$), she pays the minimum of the two, which in expectation equals

$$E[\min\{b_1, b_2\}] = \int_{\underline{b}}^{1} 2b(1-F(b))\, dF(b).$$

Putting both pieces together and using the quote distribution characterized in Proposition 1, the next lemma computes the expected price paid by a buyer conditional on search.

**Lemma 2** *Conditional on search, the expected price paid by the buyer is $\rho \in (0,1)$.*

Importantly, the Lemma above implies that expected prices conditional on search are increasing in $\rho$. The reason is two-fold. First, there is a simple *probability effect*: the higher $\rho$, the less likely it is that the buyer can compare the two quotes and choose the lowest one. And second, there is a *competition effect*: the higher $\rho$, the weaker is the competition between the sellers and hence the higher the expected quotes. Thus, expected search behavior has a direct translation on how total surplus (gross of search costs) is distributed: conditional on search, sellers make profits $\rho q$, and the buyer obtains gross utility $(1-\rho)q$.[17] Put differently, $\rho$ is a measure of the sellers' market power that allows them to extract a great fraction of the consumer's surplus.

[17]Search intensity also affects price dispersion. Indeed, the coefficient of variation (measured as the ratio between the standard deviation and the mean of prices, see Sorensen, 2000, among others) is monotonically decreasing in

So far, we have characterized equilibrium outcomes as a function of $\rho$. Interestingly, we have found that expected prices conditional on search do not depend directly on $q$. However, since $\rho$ is to be endogenously determined once we incorporate the buyer's optimal search behavior, $q$ will indirectly affect prices both through its effect on $\rho$, as well as possibly through its effect on the buyer's decision to search. To analyze this in detail, we now turn our attention to characterizing buyers' optimal search behavior.

### 3.2 Search decisions

Once the buyer has observed her realized search cost $c$, she has to decide whether to engage in search and if so, whether to search once or twice. Her utility from searching once or twice is respectively given by $u_1$ and $u_2$ (recall that the buyer's valuation for the good is normalized to 1),

$$\begin{aligned} u_1 &= (1 - E[b])\, q - c \\ u_2 &= (1 - E[\min\{b_1, b_2\}])\, q - 2c. \end{aligned}$$

The next Proposition characterizes the buyer's optimal search strategy.

**Proposition 2** *The buyer's optimal search strategy follows a cut-off rule: for given $\rho \in (0,1)$, there exist $0 < c_1(q) < c_0(q) < q$ such that the buyer does not search if $c > c_0(q)$, she searches once if $c \in (c_1(q), c_0(q)]$ or she searches twice otherwise.*

For the buyer to find it optimal to search, her utility from doing so must be non-negative, i.e., the expected gross utility from search must exceed her search cost. This amounts to

$$c \leq c_0(q) = (1 - E[b])\, q.$$

Using the distribution of price quotes characterized in Proposition 1, $c_0$ can be expressed as

$$c_0(q) = \left(1 - \frac{1}{2}\frac{\rho}{1-\rho}\ln\frac{2-\rho}{\rho}\right) q.$$

Holding $\rho$ constant, $c_0$ is increasing in $q$: *ceteris paribus*, larger buyers gain more from participating in search. To the contrary, the threshold $c_0$ is decreasing in $\rho$: a higher $\rho$ reflects higher expected prices, making search less attractive. On one extreme, if $\rho = 0$, expected prices are equal to (zero) marginal costs; hence, the buyer engages in search whenever $c \leq c_0 = q$. On the other extreme, if $\rho = 1$, expected prices are equal to the buyer's valuation. Hence, the buyer engages in search only if it is free, $c \leq c_0 = 0$. In equilibrium, since $\rho \in (0,1)$ (Lemma 1), $c_0 \in (0, q)$.

$\rho$. Hence, one should expect lower price dispersion in markets with higher prices –a result that is reminiscent of Stigler's seminal contribution despite the fact that he assumed the quote distribution to be exogenously given, i.e., independent of the intensity of search.

Consider now the decision to search twice: for it to be optimal, the buyer's search cost has to be below the expected savings from observing two rather than just one quote. This amounts to

$$c \leq c_1(q) = (E[b] - E[\min\{b_1, b_2\}])\, q.$$

Using Proposition 1, $c_1$ can be expressed as

$$c_1(q) = q - \frac{c_0(q)}{1-\rho}.$$

Holding $\rho$ fixed, $c_1$ increases in $q$, but less so than $c_0$. Unlike $c_0$, the threshold $c_1$ is non-monotonic in $\rho$ given that $\rho$ affects expected prices both when the buyer searches once as well as when she searches twice, with both effects pushing $c_1$ in opposite directions.[18]

### 3.3 Equilibrium characterization

In equilibrium, the buyer's beliefs about the distribution of quotes in the market must be consistent with sellers' actual pricing behavior. Likewise, conditional on search, sellers' beliefs about the buyer's search strategy must be consistent with her actual search behavior. Thus, in equilibrium, the following condition must satisfied:

$$\rho = 1 - G\left(c_1(q,\rho) \middle|\, c \leq c_0(q,\rho)\right),$$

where we have explicitly added the arguments $(q, \rho)$ to stress that the search thresholds $c_0$ and $c_1$ depend on the buyer's size as well as on her expected search behavior.

Importantly, since the buyer only finds it optimal to search when her search cost realization is sufficiently low, her participation decision signals that her search cost is below $c_0$. The equilibrium condition incorporates this, as the distribution that sellers use to compute the buyer's expected search behavior is truncated at $c_0$. More succinctly, the above expression can be re-written as

$$\rho = 1 - \frac{G(c_1)}{G(c_0)} \in (0,1). \tag{1}$$

Together with our previous results, the solution to equation (1) completes the characterization of the Subgame Perfect Nash Equilibrium (SPNE). The following Proposition guarantees that there always exists a solution to equation (1), which is interior (in line with Lemma 1).

**Proposition 3** *There exists a symmetric SPNE in which sellers price as stated in Proposition 1 and buyers search as stated in Proposition 2. Conditional on search, the probability that the buyer searches once is given by the solution to* (1)*.*

Furthermore, if we consider distributions that satisfy the following condition for all values of

[18] In particular, $c_1$ first increases and then decreases in $\rho$. Furthermore, $\lim_{\rho\to 0} c_1 = \lim_{\rho\to 1} c_1 = 0$. This is in line with Burdett and Judd (1983); see their Figure 1. The appendix contains the details.

$\rho \in (0,1)$,

$$\frac{g(c_0)}{G(c_0)}\frac{\partial c_0}{\partial \rho} \leq \frac{g(c_1)}{G(c_1)}\frac{\partial c_1}{\partial \rho}, \tag{2}$$

the solution is unique. This condition guarantees that the right hand side of equation (1) is everywhere decreasing in $\rho$ and hence crosses the 45-degree line only once. A sufficient (not necessary) condition for it to be satisfied is that the elasticity of the search cost distribution $G$, $\varepsilon(c) \equiv cg(c)/G(c)$,[19] is increasing in $c$. Nevertheless, these are not necessary conditions for uniqueness. If the elasticity is not everywhere increasing, a sufficient condition for uniqueness is that $G$ is not too concave.[20] The family of distribution functions for which the solution is unique is very broad, including the uniform, normal, and exponential distributions, plus all log-convex distributions, among others. In the rest of the paper, we assume that the search cost distribution is always such that the equilibrium is unique.

### 3.4 How do prices depend on the buyer's size?

We are now ready to assess whether sellers compete more or less aggressively when selling to large or small buyers. We start by focusing on the expected price conditional on search, $\rho$ (Lemma 2).

**Proposition 4** *In equilibrium, the expected discriminatory price conditional on search, $\rho^d(q)$, is (i) decreasing in $q$ if $\varepsilon(c_1^*) > \varepsilon(c_0^*)$; (ii) increasing in $q$ if $\varepsilon(c_1^*) < \varepsilon(c_0^*)$; and (iii) independent of $q$ if $\varepsilon(c_1^*) = \varepsilon(c_0^*)$.*

Sellers compete more aggressively for buyers who are expected to search more. But, is it always the case that, conditional on search, sellers expect large buyers to search more? The answer is no. To understand why, let us decompose a buyer's willingness to search in two components: the *gains from search* and the *costs of search*. On the one hand, the gains from search are higher for bigger buyers given that any potential price savings achieved through search are proportional to $q$. We refer to this as the "gains-from-search effect". On the other hand, even if all buyers have ex-ante equal expected search costs, their decisions to engage in search convey different information regarding their search costs. Precisely because small buyers stand to gain less from search, sellers expect those small buyers who engage in search to have relatively lower search costs. We refer to this as the "signalling-through-participation effect". Thus, whether sellers expect large or small buyers to search more, conditional on search, critically depends on the interplay between these two countervailing effects.[21]

[19] Note that the elasticity can also be expressed as $\varepsilon(c) \equiv cr(c)$, where $r = g/G$ is the reverse hazard rate. The elasticity is decreasing in $c$ if $\partial r/\partial c < -r/c$, i.e., the reverse hazard rate has to be sufficiently decreasing in $c$. For $G$ log-convex, the elasticity is everywhere increasing.

[20] As shown in the Appendix, this guarantees that the slope of the schedule $1 - G(c_1)/G(c_0)$ is either negative or, if positively sloped, its slope is never above 1.

[21] If the first quote is free (as in Burdett and Judd, 1983), participation is not an issue. In this case, the equilibrium condition is simply $\rho = 1 - G(c_1)$. It is easy to see that in this case a unique equilibrium always exists. Furthermore, the equilibrium comparative statics show that bigger buyers always search more and firms always compete more fiercely to serve them. Thus, the fact that the search participation decisions are endogenous is crucial to assess the comparative statics of prices as a function of consumers' size.

The "gains-from-search" and the "signalling-through-participation" effects work in opposite directions. Whether one effect or the other dominates, depends on the shape of $G$. In particular, it depends on the elasticity of the search cost distribution.[22] Suppose that the elasticity of $G(c)$ at $c_1^*$ is greater (lower) than at $c_0^*$. Hence, from the sellers' point of view, an increase in $q$ increases the probability that the buyer asks for two quotes, $G(c_1^*)$, more (less) than it increases the probability that the buyer engages in search, $G(c_0^*)$. Hence, the conditional probability that the buyer has searched only once, $\rho^*$, decreases (increases) in $q$. In sum, if the elasticity of the search cost distribution is higher (lower) at $c_1^*$ than at $c_0^*$, sellers expect that, conditional on search, large buyers are more likely to search twice than small buyers. Hence, sellers compete more (less) aggressively to serve the bigger (smaller) buyers, and charge them lower (higher) prices as a result. If the elasticity is the same at these two thresholds, the "gains-from-search" and the "signalling-through-participation" effects cancel out. Hence, sellers expect all buyers to search with the same intensity, regardless of their size.[23]

The shape of the search cost distribution ultimately determines whether $\varepsilon(c_1^*)$ is higher or smaller than $\varepsilon(c_0^*)$, and thus the comparative statics of expected prices conditional on search with respect to $q$. A sufficient condition for prices to be decreasing, increasing or constant in $q$ is that the elasticity of $G(c)$ is either decreasing, increasing or constant, respectively. There are several distribution functions with monotone elasticities. For instance, under the exponential or the Pareto distribution, elasticities decrease monotonically in $c$, implying that larger consumers pay lower prices. In contrast, if search costs are uniformly distributed with $\underline{c} < 0$ (i.e., some buyers enjoy searching), the elasticity is increasing in $c$ in the interior range (for $c > 0$), so that larger consumers pay higher prices. However, in the absence of shoppers, i.e., $\underline{c} = 0$, the elasticity is constant so that all consumers pay the same price.

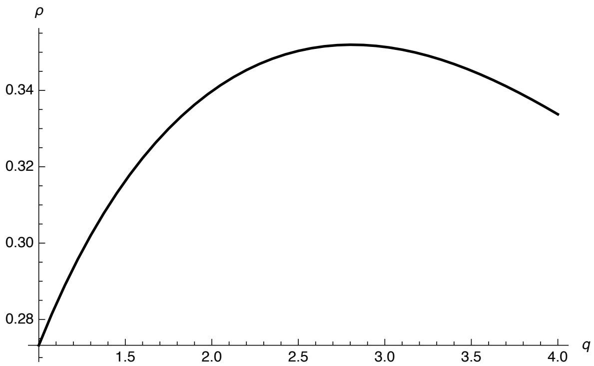
For many commonly used distributions (e.g. the Normal distribution), the elasticity is non-monotonic. However, for $q$ such that at both $c_1^*$ and $c_0^*$ the elasticity is either decreasing or increasing, the same comparative static results as above apply. In particular, if the elasticity is concave in $q$ (e.g. under the Normal distribution), prices are increasing in $q$ for small buyers (i.e., those for which $c_1^*$ and $c_0^*$ lay on the region of increasing elasticity). In contrast, prices are decreasing in $q$ for large buyers (i.e., those for which $c_1^*$ and $c_0^*$ lay on the region of decreasing elasticity). Hence, the medium-sized consumers are those who pay the highest prices: they are not small enough to benefit from signaling a low search cost when they participate in search, and they are not large enough to benefit from signaling high gains from search.

Figure 1 provides graphical examples of expected prices conditional on search under alternative distributional assumptions. One can see that prices follow a non-monotonic pattern in the case of the Normal distribution, being highest for medium-sized buyers. Prices are always decreasing in

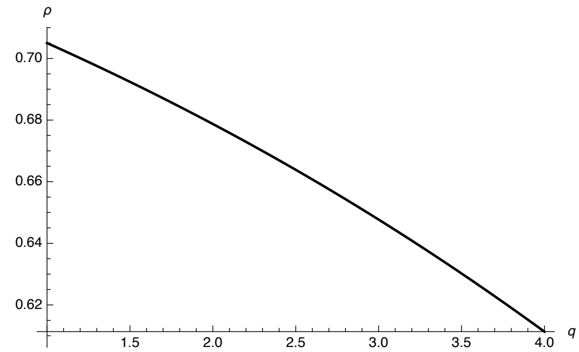
[22] There are several log-concave distribution functions for which $\varepsilon(c)$ is decreasing, e.g. the Exponential function $G(c) = \left(1 - e^{-c}\right)$ or the Pareto distribution $G(c) = 1 - \frac{1}{(c+1)^{\alpha}}$. The family of functions $G(c) = c^{\varepsilon}$ with $\underline{c} = 0$ have constant elasticity.

[23] This discussion is reminiscent of the literature in Public Finance that deals with the elasticity of earnings with respect to the tax rate over the distribution of income. See Saez (2001).

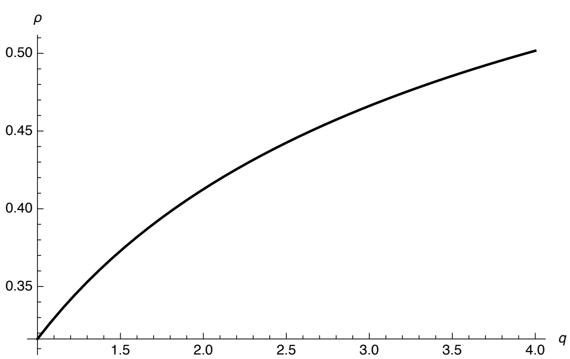
Figure 1: Expected prices (conditional on search) as a function of size $q$




(a) Non-monotonic expected prices (conditional on search) for Normal distribution, $\{\mu = 0, \sigma = 1\}$




(b) Decreasing expected prices (conditional on search) for Exponential distribution, $\{\beta = 1\}$




(c) Increasing expected prices (conditional on search) for Uniform distribution, $\{\underline{c} = -1, \overline{c} = 4\}$


(d) Flat expected prices (conditional on search) for Uniform distribution, $\{\underline{c} = 0, \overline{c} = 4\}$

*Notes:* Normal distribution as $N(\mu, \sigma)$. Exponential distribution parameterized as $CDF(c, \beta) = 1 - e^{\frac{c}{\beta}}$. Uniform distribution in range $U \sim [\underline{c}, \overline{c}]$.

size under the exponential distribution, due to the elasticity being everywhere decreasing. Finally, for the case of the uniform distribution, prices are increasing in size in the presence of shoppers, but they are constant otherwise. Indeed, when search costs are uniformly distributed between -1 and 4, i.e., with a 20% mass of shoppers, larger consumers face higher expected prices. Instead, when search costs are uniformly distributed between 0 and 4 all consumers pay the same price. This illustrates that the presence of shoppers can have a strong impact on the prices charged to the other buyers.

We can use the model to shed light on a related question: the likelihood with which large and small buyers engage in search versus taking the outside option, $G(c_0^*)$. Taking the derivative with respect to $q$,

$$\frac{\partial G(c_0^*)}{\partial q} = g(c_0^*) \left( \frac{c_0^*}{q} + \frac{\partial c_0^*}{\partial \rho^*} \frac{\partial \rho^*}{\partial q} \right). \tag{3}$$

The likelihood of search is increasing in $c_0^*$, which in turn is increasing in $q$ and decreasing in $\rho^*$. Thus, for given $\rho$, larger buyers participate in search more often relative to the smaller ones. However, $c_0^*$ also depends on $q$ through $\rho^*$. If $\rho^*$ is non-increasing in $q$ (which according to Proposition 4 occurs if $\varepsilon(c_1^*) \geq \varepsilon(c_0^*)$), then equation (3) is unambiguously positive: larger buyers engage in search more often both because they gain more from search, but also because sellers compete more to serve them. In turn, this implies that larger buyers are unambiguously better-off, relative to the smaller ones.

Matters are not as clear when $\varepsilon(c_1^*) < \varepsilon(c_0^*)$. If $\rho^*$ is increasing in $q$, the sign of equation (3) is *a priori* ambiguous (the first term in parenthesis is positive while the second one is negative). Even though we do not have a general proof for this result, the most likely outcome is that the 'direct' effect outweighs the 'indirect' effect due to higher prices, which would also imply that large buyers engage in search more often. In this case, the relationship between the buyer's size and her (ex-ante) utility remains ambiguous: even when smaller buyers obtain lower expected prices once they search, they need not be better off as compared to the large buyers as they tend to engage in search less often.

## 4 Uniform Prices

In this section we characterize equilibrium prices when sellers have to quote the same per-unit prices to all the buyers, irrespective of their size.

When price discrimination is not possible, firms choose their price quotes according to a mixed strategy that cannot be conditioned on the buyer's size. Let $\rho(q)$ denote the probability with which a buyer of size $q$ searches once only. A seller's ex-ante expected profits from pricing at $b$ are given by,

$$\pi(b) = b \int \left[ \frac{\rho(q)}{2} + (1 - \rho(q))(1 - F(b)) \right] G(c_0(q))\, q dH(q).$$

Profits are now weighted by the distribution of buyers' types in the population, $H(q)$, as well as by the probability that they engage in search, i.e., the endogenous object $G(c_0(q))$.

As shown in the following Proposition, there exists a symmetric equilibrium quote distribution which is analogous to the one in Proposition 1. The sole difference is that $\rho$ is now replaced by $\tilde{\rho}$, which reflects the weighted average of the different values of $\rho(q)$ across the population of buyers who search. In other words, firms price *as if* they were facing the "average" buyer.

**Proposition 5** *With uniform prices, there is a unique symmetric equilibrium quote distribution. It is atomless, and it is given by the same expression as in Proposition 1, with $\rho$ replaced by the weighted average of the $\rho(q)$ values in the population,*

$$\tilde{\rho} = \frac{\int \rho(q)\, G(c_0(q))\, q dH(q)}{\int G(c_0(q))\, q dH(q)} \in (0,1). \tag{4}$$

Even if all sellers quote the same prices to all buyers, the buyer-specific probability of searching once, $\rho(q)$, need not be the same across all sizes. In turn, this might introduce differences in the prices effectively paid by each buyer.[24]

For a given pricing behavior of the sellers, buyers' search decisions do not depend on whether other consumers face similar or dissimilar prices. Thus, search behavior is not directly affected by whether price discrimination is allowed or not. The effect is only indirect, to the extent that sellers' pricing behavior changes when price discrimination is no longer allowed. Accordingly, Proposition 2 remains unchanged. The sole difference is that the critical thresholds for the search strategy are now a function of the average $\tilde{\rho}$ rather than being buyer-specific. To make this explicit, we now denote the search thresholds as $\tilde{c}_1$ and $\tilde{c}_0$.

The equilibrium values of $\rho(q)$, one for each buyer type, are now found as the solution to the following system of equations, one for each $q$:

$$\rho(q) = 1 - \frac{G(\tilde{c}_1)}{G(\tilde{c}_0)} \in (0,1). \tag{5}$$

Each of these conditions is analogous to (1), with two main differences. First, there are now as many conditions as buyer sizes. And second, the right hand side of the equation now depends on the average $\tilde{\rho}$ rather than on $\rho(q)$. This implies that the search intensities that we found in the equilibrium with price discrimination cannot be an equilibrium in the game with uniform prices, unless the search intensities of all buyers are identical, i.e., unless $\rho(q) = \tilde{\rho}$ for all $q$. From the analysis in the previous section we know this is the case for distribution functions $G(c)$ with constant elasticity, e.g. the uniform distribution with $\underline{c} = 0$.

This has an important implication: when search costs are uniformly distributed with $\underline{c} = 0$, equilibrium prices and the search intensity are the same with and without price discrimination. For all other distributions, there does not exist equilibria such that all buyers use the same search intensity. Therefore, equilibrium prices change when discrimination is no longer allowed. Analogously to Proposition 3, there exists a SPNE in the game with uniform prices.

**Proposition 6** *With uniform prices, there exists a symmetric SPNE in which sellers price as stated in Proposition 5 and buyers search as stated in Proposition 2. Conditional on participating in search, the probability that the buyer asks for one quote is given by the solution to equation (5).*

Under uniform prices, the comparative statics of search intensity and expected prices with respect to buyers' size are analogous to those in Proposition 4, as shown next.

**Proposition 7** *In equilibrium, the expected uniform price conditional on search, $\rho^u(q)$, is (i) decreasing in $q$ if $\varepsilon(\tilde{c}_1^*) > \varepsilon(\tilde{c}_0^*)$; (ii) increasing in $q$ if $\varepsilon(\tilde{c}_1^*) < \varepsilon(\tilde{c}_0^*)$; and (iii) constant in $q$ if $\varepsilon(\tilde{c}_1^*) = \varepsilon(\tilde{c}_0^*)$.*

[24] Interestingly, note that the distribution of sizes (which is exogenous) affects $\tilde{\rho}$, and hence the prices charged to all buyers in the no-discrimination case. In this sense, changes in the distribution of buyers' size (e.g. following a merger between buyers) affect the degree of competition among sellers, thus affecting all buyers in the industry. The effect might be positive or negative depending on whether the average $\tilde{\rho}$ decreases or increases after the merger, an issue which will depend on the relative sizes of the merging parties.

Proposition 7 highlights that, even though all buyers face the same quote distribution, they search differently depending on their size. As a result, those buyers who solicit relatively more quotes pay lower prices. However, these are not necessarily the larger buyers. Just as before, the comparison of the elasticities at the search thresholds, $\tilde{c}_1^*$ and $\tilde{c}_0^*$, determines whether larger or smaller buyers solicit more quotes, thus giving rise to expected prices (conditional on search) that are either decreasing or increasing in consumers' size.

## 5 Price Discrimination *versus* Uniform Prices

We are now ready to understand the effects of allowing for price discrimination by comparing the results obtained in Sections 3 and 4. We will restrict attention to search cost distributions that satisfy condition (2), which guarantees that the solutions in each case are unique.

We first focus on comparing the search intensity of a buyer with size $q$.

**Proposition 8** *Assume that $G$ satisfies (2). (i) If $G$ has a decreasing elasticity, there exists $\widetilde{q}$ such that $\rho^u(q) \geq \rho^d(q) \geq \widetilde{\rho}$ for $q \leq \widetilde{q}$ (small buyers) and $\rho^u(q) < \rho^d(q) < \widetilde{\rho}$ for $q > \widetilde{q}$ (large buyers); and (ii) if $G$ has an increasing elasticity, there exists $\widetilde{\widetilde{q}}$ such that the reverse ordering holds.*

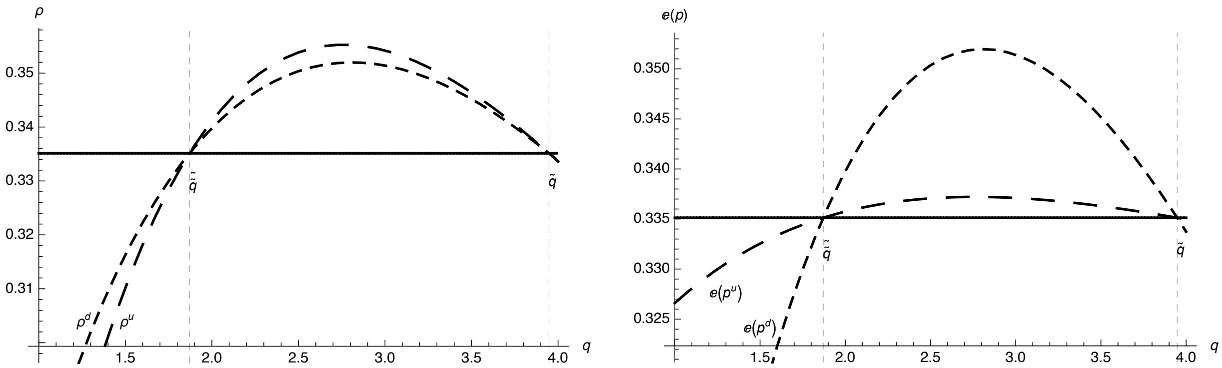
Consider the case of decreasing elasticities (part $(i)$ of the statement) and let us refer to a small (large) buyer as one with a size below (above) $\widetilde{q}$. With uniform prices, small buyers benefit from being pooled with large buyers. Hence, they can obtain lower prices than under discriminatory prices with no need to search as much (i.e., for small consumers, the probability of searching once is higher under uniform prices, $\rho^u(q) \geq \rho^d(q)$). On the contrary, large buyers do not obtain a price advantage for being large and thus need to search more relative to the case with price discrimination (i.e., for large buyers, $\rho^u(q) < \rho^d(q)$). The opposite holds true when the distribution function has increasing elasticities, as in this case the large buyers gain from being pooled with the small ones.

As stated in the following Proposition, this logic allows us to conclude on the effects (on prices, search and surplus) across consumers of banning price discrimination.

**Proposition 9** *(i) Suppose that $G$ has a decreasing elasticity. Conditional on search, the expected price paid by the small (large) buyers is lower (greater) under uniform prices than under price discrimination. Small (large) buyers also engage in search more (less) often, but conditional on search, they search less (more) intensively under uniform prices than under price discrimination. Hence, small (large) buyers are better (worse)-off under uniform prices than under price discrimination. (ii) The reverse ordering holds if $G$ has an increasing elasticity.*

Consider a search cost distribution function with decreasing elasticities. Since the price effect is first order relative to the search effect, as we move from discriminatory to uniform pricing, the prices paid by the small buyers are reduced, while those of the large buyers are increased. In turn, the lower prices faced by the small buyers induce them to engage in search more often under

Figure 2: Comparison between uniform and discriminatory pricing




(a) Probability of soliciting one quote conditional on search, Normal distribution $\{\mu = 0, \sigma = 1\}$

(b) Equilibrium expected price conditional on search, Normal distribution $\{\mu = 0, \sigma = 1\}$

*Notes:* Comparison of search intensities and equilibrium prices between uniform and discriminatory prices under the Normal distribution. The solid line represents average equilibrium search behavior ($\widetilde{\rho}$) and average expected prices under uniform prices, $\widetilde{p}^u$. The long dash line represents these same objects but conditional on $q$, i.e., $\rho(q)$ and $E[p^u(q)]$. The short dash line represents equilibrium outcomes under price discrimination, i.e., $\rho^d(q)$ and $E[p^d(q)]$. Example computed with four types with equal weight and $q \in \{1, 2, 3, 4\}$.

uniform pricing. It follows that small (large) buyers are better off under uniform (discriminatory) prices. However, the reverse ranking holds under increasing elasticities.

Figure 2 illustrates the results from Propositions 8 and 9 when search costs are normally distributed: the left panel represents search intensities and the right panel represents expected prices (conditional on search). Let us focus on the range with increasing elasticity (from low $q$s up to $\widetilde{\widetilde{q}} \approx 2$). The probability that a buyer with $q < \widetilde{\widetilde{q}}$ asks for one quote only is lower under uniform prices (long dash) than under price discrimination (short dash). However, such buyer ends up paying more than under price discrimination due to the cross-subsidization to other buyer types. The mirror image is obtained for the range with a decreasing elasticity (above $\widetilde{q} \approx 3.8$).

One can also see that the price dispersion across buyers' types is naturally smaller under uniform prices, due to the pooling of the price distribution. The attenuation in dispersion is particularly marked for those consumers on the tails of the size distribution. Under the Normal distribution, such differences get wider as one moves away from the consumer that is equally well off with uniform or with discriminatory pricing.

# 6 Conclusions

In this paper we have built a model of search that sheds light on the determinants of price discrimination across heterogeneous buyers, an issue that is becoming increasingly policy relevant.

In our model, size differences introduce heterogeneity in buyers' willingness to search, which opens up the door for price discrimination. In particular, sellers charge lower prices to those buyers with a higher perceived willingness to search. The key to assessing buyer power is thus to understand whether large buyers are perceived to have higher or lower willingness to search (conditional on search) as compared to the small ones. In this paper we have shown that this issue ultimately depends on the shape of the search cost distribution function.

In particular, we have identified a simple condition to predict the relationship between prices and buyers' size: if the elasticity of the search cost distribution is decreasing (increasing) over the relevant range, than larger buyers pay less (more) then the smaller ones. If search costs are normally distributed, the elasticities depict an-inverted U-shaped. Hence, prices conditional on search are also non-monotonic in size, with medium-size consumers paying more than either the small or the large ones.

Our analysis provides predictions regarding the effects of price discrimination on the various consumers. Since these predictions rely on the elasticities of the search cost distribution, which need not coincide across all markets, it is simply not possible to provide a single answer to the general question of how a ban on price discrimination would affect consumers' prices. Inexorably, the answer has to rely on industry specific studies that shed light on consumers' search behavior as well as on the distribution of the various consumers' types in the market.

We have made some simplifying assumptions. For instance, we have assumed that consumers size and search costs are independent. In contrast, if larger buyers were more efficient in searching, their prices would go down relative to the prices faced by the small buyers. For instance, if search costs were uniformly distributed, consumers would no longer face equal prices. Instead, larger buyers would face lower prices, just as in the case with decreasing elasticities. To highlight that price differences arise due to search frictions alone, we have also assumed that all consumers have equal valuations. If consumers differed in their size and in their valuation, equilibrium pricing would be affected but results would still depend on the elasticity of the search cost distribution. For instance, with increasing elasticities, if bigger buyers valued the good more, they would face higher prices for a two-fold reason: their weaker perceived willingness to search and their higher valuation. With decreasing elasticities, the two effects would point in opposite directions, implying that the final outcome would depend on which of the two effects dominated.

Our model should prove helpful to understand price discrimination in imperfectly competitive markets due to search frictions, and thus provide testable predictions for empirical work in this area.

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# A Appendix: Proofs

**Proof of Lemma 1.** Suppose $\rho = 1$. Then, the seller knows that it is a monopolist and hence charges the reservation price. However, it would leave no surplus for the buyer, so her participation constraint would not be satisfied. Hence, $\rho = 0$ cannot be part of an equilibrium in which the buyer has decided to participate. Suppose $\rho = 0$. Then, there is Bertrand competition with both sellers quoting prices equal to marginal cost. However, knowing that all sellers choose the same quote, the buyer would have incentives to deviate and search less in order to save on search costs. Hence, $\rho = 0$ cannot be part of an equilibrium. As a consequence, in equilibrium, we must have $\rho \in (0, 1)$.

■

**Proof of Proposition 1.** Let sellers choose the distribution of quotes $F(b)$ over the interval $\left[\underline{b}, \overline{b}\right]$ . Standard arguments imply that this distribution must be atomless. In particular, if sellers put mass on a given quote, there would be a positive probability of a tie. If such a quote is above marginal costs, each seller would be better off by slightly reducing its quote below that level: this would have a minor effect on its profits if the buyer has asked for one quote only, but would guarantee that the seller makes the sale whenever the buyer has asked for two quotes. Putting mass at marginal cost cannot be part of an equilibrium either, given that each seller would be able to make the sale at a higher price whenever the buyer has asked for one quote only.

Seller $i$'s profits from quoting a price $b$ when rivals are choosing quotes according to $F(b)$ are given by:[25]

$$\pi(b) = bq\left(\frac{\rho}{2} + (1-\rho)(1-F(b))\right).$$

Since all the quotes in the support of the mixed strategy equilibrium must yield the same expected profits, it follows that

$$\pi(b;q) = \overline{\pi} \text{ for all } b \in \left[\underline{b}, \overline{b}\right].$$

Furthermore, as $\pi\left(\overline{b}\right) = \overline{b}q\frac{\rho}{2}$ is increasing in $\overline{b}$, it follows that $\overline{b} = 1$. Hence, $\overline{\pi} = q\frac{\rho}{2}$. From $\pi(\underline{b}) = \underline{b}q\left(\frac{\rho}{2} + (1-\rho)\right) = \overline{\pi} = q\frac{\rho}{2}$ it also follows that $\underline{b} = \frac{\rho}{2-\rho}$. Accordingly, the support of the equilibrium mixed strategy is $b \in \left[\frac{\rho}{2-\rho}, 1\right]$.

To obtain the equilibrium quote distribution, from $\pi(b) = q\frac{\rho}{2}$, it follows that

$$F(b) = 1 - \frac{1}{2}\frac{\rho}{1-\rho}\frac{1-b}{b},$$

with density

$$f(b) = \frac{1}{2}\frac{\rho}{1-\rho}\frac{1}{b^2}.$$

For $\rho \to 1$, the equilibrium collapses to the reservation price, whereas for $\rho \to 0$, the equilibrium collapses to marginal costs. Sellers' expected profits when quoting a price to a buyer of size $q$ are thus $q\rho$. ■

**Proof of Lemma 2 .** We first derive the distribution of the price paid by the buyer (conditional on search), which we denote as $F_p$, using the quote distribution characterized above. When the buyer has searched $n$ times, the distribution is $(1-(1-F(b))^n)$. Hence, given that buyers only get one quote with probability $\rho$, one finds the distribution the expected price as,

$$\begin{aligned} F_p(b) &= \rho F(b) + (1-\rho)\left(1-(1-F(b))^2\right) \\ &= \frac{(\rho - b(2-\rho))(\rho - b(2+\rho))}{(2b)^2(1-\rho)}, \end{aligned}$$

with density

$$f_p(b) = \frac{\rho^2}{2b^3(1-\rho)}.$$

[25] Results would be the same if we instead computed the firm's expected profits using probabilities conditional on having received a quote request. Let $x = \frac{\rho/2}{1-\rho/2}$ be the conditional probability of receiving a quote request from a buyer who has only asked for one quote only. Expected profits would thus be written as $\pi(b) = bq(x + (1-x)(1-F(b)))$. The equilibrium price distribution would be $F(b) = 1 - \frac{x}{1-x}\frac{1-b}{b}$, with $b$ in the compact support $p \in [x, 1]$. While both formulations are equivalent, we believe that using the unconditional probability $\rho$ allows for a more intuitive interpretation of the results.

Note that the price distribution $F_p(b)$ is decreasing in $\rho$,

$$\frac{\partial F_p(b)}{\partial \rho} = -\frac{\rho(2-\rho)(1-b)(1+b)}{4b^2(1-\rho)^2} < 0.$$

With this expression we can compute the expected price paid by the buyer (conditional on search) as

$$\int_{\underline{b}}^{\overline{b}} b dF_p(b) = \rho.$$

■

**Proofs of Proposition 2.** A buyer who searches once or twice derives utility

$$\begin{aligned} u_1 &= \left(1 - \int_{\underline{b}}^{\overline{b}} b dF(b)\right) q - c \\ u_2 &= \left(1 - \int_{\underline{b}}^{\overline{b}} 2b(1-F(b)) dF(b)\right) q - 2c, \end{aligned}$$

with $F(b)$ and $[\underline{b}, \overline{b}]$ as characterized in Proposition 1.

We want to find the expressions for $c_0$ and $c_1$, which are implicitly defined by $u_1(c_0) = 0$ and $u_1(c_1) = u_2(c_1)$. For this purpose, we will use the equilibrium quote distribution characterized in Proposition 1.

Conditional on observing one quote, the expected utility is

$$\begin{aligned} u_1 &= \left(1 - \int_{\underline{b}}^{\overline{b}} b dF(b)\right) q - c \\ &= q\left(1 - \frac{1}{2}\frac{\rho}{1-\rho}\ln\left(\frac{2-\rho}{\rho}\right)\right) - c. \end{aligned}$$

Hence, to be indifferent between searching or not:

$$c_0 = q\left(1 - \frac{1}{2}\frac{\rho}{1-\rho}\ln\left(\frac{2-\rho}{\rho}\right)\right).$$

The threshold $c_0$ is decreasing in $\rho$, with $\lim_{\rho\to 0} c_0 = q$ and $\lim_{\rho\to 1} c_0 = 0$. For future reference,

$$\lim_{\rho\to 1}\frac{\partial c_0}{\partial \rho} = -q < 0.$$

Conditional on observing two quotes, the expected price is the minimum of the two, so the utility is

$$
\begin{aligned}
u_2 &= \left(1-\int_{\underline{b}}^{\bar{b}} 2b\left(1-F\left(b\right)\right)dF\left(b\right)\right)q-2c \\
&= q\left(1+\frac{\rho^2}{2\left(1-\rho\right)^2}\left(2-\frac{2}{\rho}+\ln\frac{2-\rho}{\rho}\right)\right)-2c.
\end{aligned}
$$

Equating the two utilities and solving for $c$:

$$c_1 = q\frac{\rho}{1-\rho}\left(\frac{1}{2\left(1-\rho\right)}\ln\left(\frac{2-\rho}{\rho}\right)-1\right)\cdot$$

Note that we can also write it as

$$c_1 = q-\frac{c_0}{1-\rho}\cdot$$

The threshold $c_1$ is concave in $\rho$,[26] with $\lim_{\rho\to 0} c_1 = \lim_{\rho\to 1} c_1 = 0$. For future reference,

$$\lim_{\rho\to 1}\frac{\partial c_1}{\partial\rho} = -\frac{q}{3}<0\cdot$$

Taking the difference between the two thresholds:

$$c_0-c_1 = q\frac{2-2\rho-\rho\left(2-\rho\right)\ln\left(\frac{2-\rho}{\rho}\right)}{2\left(1-\rho\right)^2}\cdot$$

The difference is decreasing in $\rho$, with $\lim_{\rho\to 0}\left(c_0-c_1\right) = q$ and $\lim_{\rho\to 1}\left(c_0-c_1\right) = 0$. Wrapping up, all this shows that $0<c_1<c_0<q$ such that (i) $0>u_1\left(c\right)>u_2\left(c\right)$ for $c>c_0$; (ii) $u_1\left(c\right)\geq\max\left\{u_2\left(c\right),0\right\}$ for $c\in\left(c_1,c_0\right]$ and (iii) $u_2\left(c\right)\geq u_1\left(c\right)>0$ for $c\leq c_1$.

For future reference,

$$\frac{\partial c_0}{\partial\rho}-\frac{\partial c_1}{\partial\rho} = q\frac{2-2\rho-\ln\left(\frac{2-\rho}{\rho}\right)}{\left(1-\rho\right)^3}<0. \tag{6}$$

Also for future reference, let us note that the following inequality is always satisfied

$$\frac{c_1}{c_0}>\frac{\frac{\partial c_1}{\partial\rho}}{\frac{\partial c_0}{\partial\rho}}\cdot \tag{7}$$

■

**Proof of Proposition 3.** We need to show that there exists a solution to (1) in $(0,1)$. To show that condition (1) has a solution, note that $\lim_{\rho\to 0} c_1 = 0$ and $\lim_{\rho\to 0} c_0 = q$. Hence, when $\underline{c}<0$

$$\lim_{\rho\to 0}\left(1-\frac{G\left(c_1\right)}{G\left(c_0\right)}\right) = 1-\frac{G\left(0\right)}{G\left(q\right)}\in\left(0,1\right),$$

[26] The maximum is achieved at $\rho = 0.365$.

whereas when $\underline{c} = 0$,

$$\lim_{\rho\to 0}\left(1 - \frac{G(c_1)}{G(c_0)}\right) = 1.$$

Furthermore, $\lim_{\rho\to 1} c_1 = \lim_{\rho\to 1} c_0 = 0$, implying that $G(c_1) = G(c_0)$. Hence, when $\underline{c} < 0$

$$\lim_{\rho\to 1}\left(1 - \frac{G(c_1)}{G(c_0)}\right) = 0.$$

When $\underline{c} = 0$, both $G(c_1)$ and $G(c_0)$ take the value 0 for $\rho \to 1$, so $\lim_{\rho\to 1} \frac{G(c_1)}{G(c_0)}$ is undefined. Applying l'Hôpital,

$$\begin{aligned}
\lim_{\rho\to 1}\left(1 - \frac{G(c_1)}{G(c_0)}\right) &= 1 - \lim_{\rho\to 1} \frac{g(c_1)\frac{\partial c_1}{\partial \rho}}{g(c_0)\frac{\partial c_0}{\partial \rho}} \\
&= 1 - \frac{g(0)}{g(0)}\frac{q/6}{q/2} \\
&= \frac{2}{3}.
\end{aligned}$$

Under regularity conditions on $G$ that ensure continuity, note that: (i) for $\underline{c} < 0$, the function takes a strictly positive value for $\rho \to 0$ and zero for $\rho \to 1$; and (ii) for $\underline{c} = 0$, the function takes the value 1 for $\rho \to 0$ and a lower value for $\rho \to 1$. Hence, in both cases, the function must cross the 45 degree line at some $\rho^* \in (0, 1)$. Hence, there exists an interior solution to (1).

[Uniqueness] A sufficient condition for equilibrium uniqueness is that $G$ is log-convex as in this case the RHS of equation (1) is everywhere decreasing in $\rho$. Hence, it crosses the 45-degree line only once. In detail, the general derivative for the function is

$$\frac{\partial\left(1 - \frac{G(c_1)}{G(c_0)}\right)}{\partial \rho} = \frac{g(c_0)\,G(c_1)\,\frac{\partial c_0}{\partial \rho} - g(c_1)\,G(c_0)\,\frac{\partial c_1}{\partial \rho}}{(G(c_0))^2}.$$

Since the denominator is always positive, we focus on the numerator. It will be negative as long as equation (2) is satisfied. Using our previous results,

$$\frac{\partial c_0}{\partial \rho} < 0 \text{ and } \frac{\partial c_0}{\partial \rho} < \frac{\partial c_1}{\partial \rho},$$

we can re-write equation (2) as

$$\frac{\frac{g(c_0)}{G(c_0)}}{\frac{g(c_1)}{G(c_1)}} > \frac{\frac{\partial c_1}{\partial \rho}}{\frac{\partial c_0}{\partial \rho}},$$

which guarantees that the RHS of equation (1) is everywhere decreasing in $\rho$. A sufficient condition for this to be satisfied is that the elasticity of $G$ is constant or increasing. In particular, if the elasticity is increasing in $c$,

$$\frac{g\left(c_0\right)}{G\left(c_0\right)}c_0 > \frac{g\left(c_1\right)}{G\left(c_1\right)}c_1 \Leftrightarrow \frac{\frac{g(c_0)}{G(c_0)}}{\frac{g(c_1)}{G(c_1)}} > \frac{c_1}{c_0} > \frac{\frac{\partial c_1}{\partial \rho}}{\frac{\partial c_0}{\partial \rho}},$$

where the last inequality follows from equation (7).

If the elasticity of $G$ is decreasing, the right hand side of equation (2) is decreasing in $\rho$ up to $\rho \leq 0.365$ as in this case $\partial c_1/\partial \rho > 0$ . However, for $\rho > 0.365$ the right hand side of equation (1) can eventually become positively sloped. We cannot then resort to the same argument as before to show uniqueness.

A less stringent sufficient condition for uniqueness is that the slope of the right hand side of (1) is below 1. This condition becomes

$$\frac{G\left(c_0\right)}{G\left(c_1\right)} > \frac{g\left(c_0\right)}{G\left(c_0\right)}\frac{\partial c_0}{\partial \rho} - \frac{g\left(c_1\right)}{G\left(c_1\right)}\frac{\partial c_1}{\partial \rho}.$$

If condition (2) is satisfied, the RHS of the above equation is negative so that this condition is always satisfied. If the RHS is positive (meaning that the schedule $1 - G\left(c_1\right)/G\left(c_0\right)$ is positively sloped), this condition essentially requires that $G$ is not too concave. ■

**Proof of Proposition 4.** For $\rho^d$ to be decreasing (increasing) in $q$, the RHS of the equilibrium condition (1) must be decreasing (increasing) in $q$. This is the case if and only if

$$\frac{\partial\left(1 - \frac{G(c_1)}{G(c_0)}\right)}{\partial q} = -\frac{c_1 G\left(c_0\right) g\left(c_1\right) - c_0 G\left(c_1\right) g\left(c_0\right)}{q\left(G\left(c_0\right)\right)^2} < 0.$$

where we have used the fact that both $c_1$ and $c_0$ are linear in $q$ so that their derivatives are simply $c_1/q$ and $c_0/q$ respectively. Since the denominator is positive, we focus attention on the numerator, which can be re-written as

$$G\left(c_0\right)G\left(c_1\right)\left[c_0\frac{g(c_0)}{G(c_0)} - c_1\frac{g(c_1)}{G(c_1)}\right] < 0,$$

or using the expression for $\varepsilon\left(c\right) \equiv c\frac{g(c)}{G(c)}$,

$$G\left(c_0\right)G\left(c_1\right)\left[\varepsilon\left(c_0\right) - \varepsilon\left(c_1\right)\right] < 0.$$

It follows that a sufficient condition for $\rho^d$ to be decreasing (increasing) in $q$ is that the term in brackets, evaluated at $\rho^d$, is negative (positive). Hence, as $q$ goes up, the schedule crosses the 45-degree line at a smaller (higher) value of $\rho$. If the elasticity $\varepsilon\left(c\right)$ is constant, changes in $q$ do not move the schedule, and hence the equilibrium remains unchanged. Note that if $G$ is log-convex, then $\varepsilon\left(c_0\right) > \varepsilon\left(c_1\right)$ so that $\rho^d$ is increasing in $q$. ■

**Proof of Proposition 5.** Similar arguments are those in Proposition 1 allow us to conclude that there does not exist an equilibrium in pure strategies.

Let seller $i$'s profits from quoting a price $b$ when the rival is choosing quotes according to $F(b;q)$ be given by:

$$\pi(b) = b\int q\left[\rho(q)/2 + (1-\rho(q))(1-F(b))\right]G(c_0(q))\,qdH(q).$$

Let $x = \int q\rho(q)\,G(c_0(q))\,dH(q)$ and $y = \int qG(c_0(q))\,dH(q)$ with $x < y$. Profits can be re-written as

$$\pi(b) = b\left(x + (y-x)(1-F(b))\right).$$

Since all the quotes in the support of the mixed strategy equilibrium must yield the same expected profits, it follows that

$$\pi(b) = \overline{\pi} \text{ for all } b \in \left[\underline{b}, \overline{b}\right].$$

Furthermore, as $\pi\left(\overline{b}\right) = \overline{b}y$ is increasing in $\overline{b}$, it follows that $\overline{b} = 1$. Hence, $\overline{\pi} = x$. From $\pi(\underline{b}) = \underline{b}y = \overline{\pi} = x$ it also follows that $\underline{b} = \frac{x}{y}$. To obtain the equilibrium quote distribution, from $\pi(b) = x$, it follows that

$$F(b) = 1 - \frac{1}{2}\frac{x}{y-x}\frac{1-b}{b}\cdot$$

Renormalize it by defining

$$\tilde{\rho} \equiv \frac{\int \rho(q)\,qG(\tilde{c}_0(q))\,dH(q)}{\int qG(\tilde{c}_0(q))\,dH(q)} < 1$$

(i.e., $\tilde{\rho}$ is the weighted average $\rho$, and $\tilde{c}_0(q)$ and $\tilde{c}_1(q)$ are the thresholds evaluated at $\tilde{\rho}$) so that

$$F(b) = 1 - \frac{1}{2}\frac{\tilde{\rho}}{1-\tilde{\rho}}\frac{1-b}{b}\cdot$$

■

**Proof of Proposition 6.** Using the expression for $\tilde{\rho}$ above, and plugging it in the equilibrium condition,

$$\tilde{\rho} = \frac{\int\left(1 - \frac{G(\tilde{c}_1(q))}{G(\tilde{c}_0(q))}\right)G(\tilde{c}_0(q))\,qdH(q)}{\int G(\tilde{c}_0(q))\,qdH(q)}\cdot$$

With some algebra,

$$\begin{aligned}\tilde{\rho} &= \frac{\int\left(G(\tilde{c}_0(q)) - G(\tilde{c}_1(q))\right)qdH(q)}{\int G(\tilde{c}_0(q))\,qdH(q)} \\ &= 1 - \frac{\int G(\tilde{c}_1(q))}{\int G(\tilde{c}_0(q))}dH(q)\cdot\end{aligned}$$

The equilibrium conditions on the buyers' and sellers' sides can be combined into a single equilibrium equation:

$$\tilde{\rho} = 1 - \frac{\int G(c_1(q,\tilde{\rho}))}{\int G(c_0(q,\tilde{\rho}))}dH(q).$$

Therefore, one can solve the above equation to find the equilibrium $\tilde{\rho}$ by substituting in the values of $\tilde{c}_0(q)$ and $\tilde{c}_1(q)$ as a function of $\tilde{\rho}$, which is analogous to the case with price discrimination. The remainder of the proof is thus analogous to the one of Proposition 3. ■

**Proof of Proposition 7.** The proof follows similar steps as the proof of Proposition 4, and it is therefore omitted. ■

**Proof of Proposition 8.** First, we want to show that there exists at least one $\widetilde{q}$ such that at $\widetilde{q}$, $\rho^d(\widetilde{q}) = \widetilde{\rho} = \rho^u(\widetilde{q})$. These $\rho$ values are defined in equation equation (2) for $\rho^d(q)$, equation (4) for $\widetilde{\rho}$, and equation (5) for $\rho^u(q)$. The schedules $\widetilde{\rho}$ and $\rho^u(q)$ have to cross since $\widetilde{\rho}$ is the average of $\rho^u(q)$. Let $\widetilde{q}$ be the value(s) at which they cross. In turn, the RHS of the expressions for $\rho^d(q)$ and $\rho^u(q)$ are the same, so they are equal if we evaluate them at the same $(q, \rho)$. It follows that $\rho^d(\widetilde{q}) = \widetilde{\rho} = \rho^u(\widetilde{q})$.

Second, for the case of decreasing elasticities, we want to show that if $q < \widetilde{q}$ then $\rho^u(q) > \rho^d(q) > \widetilde{\rho}$; while if $q > \widetilde{q}$ then $\rho^u(q) < \rho^d(q) < \widetilde{\rho}$. The reverse ordering is true for the case of increasing elasticities.

From our previous Propositions, since $\rho^u(q)$ and $\rho^d(q)$ are decreasing in $q$, for buyers with a size below the average, $q < \widetilde{q}$, we must have $\rho^u(q) > \widetilde{\rho}$ and $\rho^d(q) > \widetilde{\rho}$. Furthermore, whenever $1 - \frac{G(c_1)}{G(c_0)}$ is decreasing in $\rho$ (as guaranteed by condition (2); see the proof of Proposition 3), the latter implies

$$\rho^u(q) = 1 - \frac{G(c_1(q, \widetilde{\rho}))}{G(c_0(q, \widetilde{\rho}))} > 1 - \frac{G\left(c_1(q, \rho^d)\right)}{G(c_0(q, \rho^d))} = \rho^d(q).$$

The proof for $q > \widetilde{q}$ as well as the proof for the case with increasing elasticities are analogous, and therefore omitted. ■

**Proof of Proposition 9.** We show the results for the case of decreasing elasticities, as those for increasing elasticities are analogous. The proof follows directly from the previous Proposition. In particular, since $\rho^d(q) > \widetilde{\rho}$ for $q < \widetilde{q}$, the small buyers pay lower prices (conditional on search) under uniform prices than under price discrimination. Since $c_0$ is decreasing in the expected price (conditional on search), and the likelihood of search $G(c_0)$ is increasing in $c_0$, it follows that small buyers engage in search more often under uniform prices than under price discrimination. Last, since $\rho^u(q) > \rho^d(q)$ for $q < \widetilde{q}$, it follows that, conditional on search, small buyers incur less search costs under uniform prices than under price discrimination. These three results together imply that small buyers are better off under uniform prices than under price discrimination. ■