Complementary Bidding Mechanisms and Startup Costs in Electricity Markets*

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Stanford GSB and NBER

January 2014

Abstract

I extend multi-unit auction estimation techniques to a setting in which firms can express cost complementarities over time. In the context of electricity markets, I show how the auction structure and bidding data can be used to estimate these complementarities, which in these markets arise due to startup costs. I find that startup costs are substantial and that taking them into account helps better explain firm bidding strategies and production patterns. As in other dynamic settings, I find that startup costs limit the ability of firms to change production over time, exacerbating fluctuations in market prices. These fluctuations can induce estimates of market power that ignore dynamic costs to overstate markup volatility, with predicted markups that can be even negative in periods of low demand. I show how accounting for startup costs can provide a natural correction for these markup biases.

JEL: L13, L94, D44.

Keywords: Auctions with complementarities, electricity markets, startup costs, market power.

*This paper is part of my thesis chapter "The Welfare Effects of Complementary Bidding Mechanisms." I am indebted to Nancy Rose and Stephen Ryan for their invaluable guidance and advice. I am grateful to Paul Joskow for his comments, support and encouragement. I also would like to thank Hunt Allcott, Lanier Benkard, Severin Borenstein, Jim Bushnell, Lucas Davis, Natalia Fabra, Meredith Fowlie, Jakub Kastl, Erin Mansur, Marti Mestieri, Steve Puller, Ignacio Pérez-Arriaga, Peter Reiss, Paul Scott, Michael Whinston, Matthew White, Frank Wolak, Catherine Wolfram and Ali Yurukoglu, researchers at the UC Energy Institute and seminar participants at various conferences and universities for their comments and suggestions. All remaining errors are my own. I acknowledge financial support from the Ramon Areces Foundation and the George and Obie Shultz Fund.

1 Introduction

Auctions are used to allocate goods in many markets. Among the most commonly studied auction settings, there are first and second price auctions for single goods, and discriminatory and uniform price auctions for multiple goods (Athey and Haile, 2007). Whereas most of the literature has focused on the study of these auction formats, our understanding of more complex mechanisms is still limited. However, in many applications, the auction process departs from these simple rules. Auctions are often augmented with rules that couple complementary bidding mechanisms with more traditional designs.

The introduction of complementary bidding mechanisms is often motivated by the presence of complementarities across goods. Additional bidding mechanisms are introduced to allow participants to express their preferences more explicitly. A well known example of such auctions are combinatorial auctions, in which bidders not only bid on single goods, but also on combinations of goods known as "packages". Such procedures are used in several procurement auctions, e.g. for spectrum and transportation.[1]

In the energy sector, many wholesale electricity markets use complementary bidding mechanisms to allow firms to express their intertemporal cost complementarities, which arise due to the presence of startup costs.[2] Some examples of markets with such mechanisms are the Pennsylvania-New Jersey-Maryland (PJM) market, the Californian market, the Irish market and the Spanish market. Even though the specifics of each mechanism can vary, they all share the same feature: they extend traditional auction formats to allow firms to express their startup costs more explicitly.

Electricity markets provide a unique environment in which to analyze firm behavior in the presence of dynamic costs. The richness and high frequency of the bidding data allows to observe not only equilibrium outcomes, but also firms' detailed strategies. Furthermore, it presents the advantage that dynamic decisions (startup) take place at a higher frequency (daily) than in other applied settings. Using a model of strategic bidding as a framework to interpret the data, I show how to elicit firms' dynamic costs exploiting the full richness of the data.

These markets also create an opportunity to study the interaction of dynamic costs and market power. Similar to other markets, dynamic costs constrain the ability of firms to modify their output levels over time. This has important implications on how markups are measured. Typical markup estimates that ignore dynamic costs will tend to overstate markup volatility. Through the lens of a static framework, a firm would appear to be producing "too much" when demand is low, and "too little" at periods of high demand, exaggerating the apparent volatility of markups. The methods that I develop provide a correction for these markup calculations, which are an important object in

[1] See Cramton, Shoham and Steinberg (2006) for a comprehensive treatment of tools and applications.

[2] Startup costs are a fixed cost incurred when a unit is turned on. It refers to costs uncurred to warm up a power plant before it can safely produce electricity.

market power analysis.

The major contributions of the paper are twofold. First, I extend the estimation of multi-unit auctions by adapting current techniques to a setting with augmented forms of bidding and dynamic costs. I show how these augmented bids can be used to identify cost complementarities in the production function of the firms. To my knowledge, this is the first paper that exploits a complementary auction mechanism to identify startup costs or, more broadly, fixed costs that generate dynamic complementarities. Second, in the context of electricity markets, this is the first paper to structurally estimate startup costs in the presence of market power. I show that startup costs help reconcile observed bidding strategies and production patterns, and how it provides a correction for market power estimates.

In this paper, I study the properties of the complementary bidding mechanism used in the Spanish wholesale electricity market. The mechanism used in the Spanish electricity market takes the form of an augmented set of uniform price auctions.[3] For every hour of the day, firms submit offers to produce electricity with increasing step bids for each production unit, as they would in a uniform price auction. These bids are called *simple bids*. In addition, each unit can also express a variable and a fixed cost component that needs to be recovered within the day, defining an implicit daily revenue requirement, which constitutes its *complex bid*. This minimum revenue requirement makes the simple bids contingent: if the daily gross revenue of a generator is not at least as large as its minimum requirement, its hourly simple bids are taken out from the auction and the generator is not assigned any quantity. Because the revenue requirement applies to the whole day, the mechanism allows firms to express their preference regarding joint realizations of demand over the day.

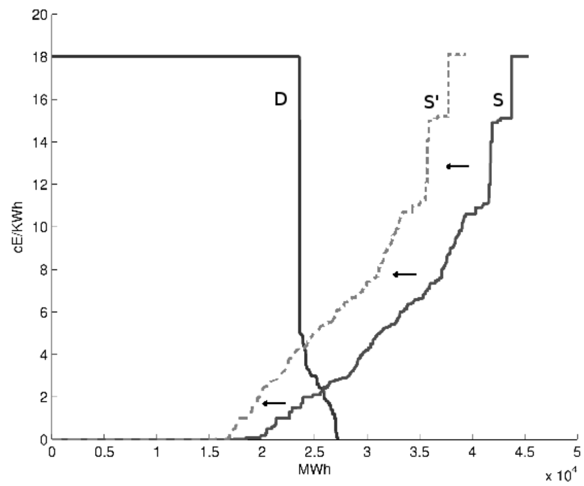
Figure 1 illustrates the effects of complex bids on the market supply curve. The figure plots demand and supply for a given hour of the day. The solid supply curve S represents the original simple bids made by the firms. Yet, at the original hourly prices defined by the crossing of the two solid lines, several units do not recover their revenue requirement. These units are taken out iteratively from the aggregate supply curve, shifting the supply curve inwards to S', until the price is such that the market clears and all minimum revenue requirements are satisfied.

I develop a multi-unit auction model to understand the impact of such auction design on firm behavior. In the model, I take into account the non-convex nature of the production function as well as its short-run dynamics, which are essential elements affecting optimal bidding. The model allows me to estimate marginal production costs and startup costs using the first order conditions implied by firm profit-maximizing behavior.

In a first stage, I use simple bids to estimate marginal costs and financial contracts (also known

[3] In a uniform price auction, the auctioneer crosses demand and supply. The market price is determined by the intersection of the two. All supply units with prices lower or equal to the market price are scheduled to produce.

Figure 1: A uniform auction in which offers are discarded



The augmented bidding procedure used in the Spanish electricity market discards those units for which the minimum revenue requirement is not satisfied, shifting the supply curve to the left (S to S').

as forward contracts), which are unobserved to the econometrician.[4] Whereas previous literature had focused on identifying either marginal costs (Wolak, 2007) or financial contracts (Hortaçsu and Puller, 2008; Allcott, 2012), I develop a strategy that allows to estimate both of them jointly. The identification strategy relies on the fact that, whereas marginal costs are realized at the power plant level, forward contracts are common at the firm level.

In a second stage, I use complex bids to identify startup costs. In line with the auction literature, the information contained in the bidding data can be useful in estimating valuations, compared to the case in which only revealed outcomes (i.e. only price or output data) are observed.[5] Due to the sealed bid format of the auction, by which all offers (winning or not) are observed, the contingent nature of the minimum revenue requirement allows the econometrician to observe what the firm would have liked to do under alternative uncertainty realizations, even if only one outcome is observed ex-post. This is particularly useful in the case of discrete choices, like starting up, that might not happen very frequently in practice.

Finally, once the fundamentals of the model are obtained, I conduct policy experiments to understand the interaction between market power and cost complementarities. I show that accounting for startup costs can help better explain the behavior of strategic firms. I also show that the introduction of startup costs in the structural model can help reconcile strategic markups across hours,

[4]Financial contracts are an important institutional feature of electricity markets. See section 2 for details.

[5]For single good auctions, Athey and Haile (2002) show how bidding data can be used to relax the assumptions needed to ensure identification if only revealed outcomes are observed. For multi-unit auctions, Hortaçsu and McAdams (2010) show that, under some assumptions, the distribution of valuations can be non-parametrically estimated even with data from a single auction.

which had been shown to exhibit a downward bias during night hours (Bushnell, Mansur and Saravia, 2008; Mansur, 2008). Finally, I show that the presence of startup costs limits the ability of a strategic firm to price discriminate across hours, with implied markups that are smoother than those obtained with a model without startup costs.

The paper is mostly related to two streams of research: the empirical auctions literature and the market power literature in wholesale electricity markets. The paper follows the methods in the empirical auctions literature to estimate valuations from underlying bidding data using implied optimality conditions (Guerre, Perrigne and Vuong, 2000). It is closely related to work in the context of to multi-unit auctions (McAdams, 2008; Hortaçsu and McAdams, 2010; Gans and Wolak, 2008; Kastl, 2011), extending estimation techniques to a setting with complementary bidding mechanisms and dynamic costs.

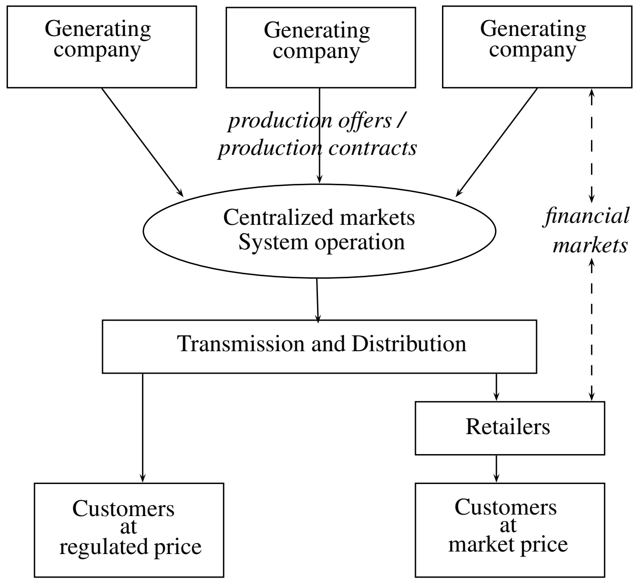
The bidding model and first-order conditions that I derive are related to previous work on wholesale electricity markets (Wolak, 2000, 2003, 2007; Hortaçsu and Puller, 2008; Allcott, 2012). The cost structure that I use is similar to the one used in Wolak (2007), to which I incorporate startup costs. The paper is also related to previous studies that have considered the role of startup decisions in competitive markets (Fowlie, 2010; Cullen, 2012*a*,*b*). Mansur (2008) pointed out that ignoring dynamic costs could severely bias market power estimates and welfare analysis. This paper proposes a structural methodology to compute market power estimates in the presence of dynamic costs.

The rest of the paper is organized as follows. Section 2 describes the institutional features of the Spanish electricity market and the data. Section 3 develops a multi-unit auction model with complex bidding and derives optimality conditions. Section 4 presents the estimation strategy and results. Section 5 details the policy experiments used to understand the interaction of dynamic costs and the exercise of market power. Section 6 concludes.

2 Institutions and Data

The Spanish electricity market is a national market that produces between 15,000 and 45,000MWh hourly, with around 85,000MW of installed capacity, serving more than 40 million people and having an annual economic volume around 8B€. Similar to other electricity markets, it consists of several important segments: generation, transmission, distribution and retailing, as represented in Figure 2. Generating firms can sell their electricity either in centralized markets or by means of production contracts. Firms can also use financial contracts to hedge price risks. Independently of how the produced quantity is financially settled, all production decisions need to be centralized to ensure the functioning of the overall system. The electricity is then delivered to final consumers

Figure 2: Market structure



by distributors and retailers.

The generating companies in the market make two main decisions. First, they decide their financial position, usually weeks or months in advance, by means of financial contracts. Financial contracts, also known as hedge contracts or contracts for differences, are firm-specific and imply that a certain amount of produced electricity is hedged and, therefore, not subject to the market price. These contracts avoid the risk implied by uncertain market prices.

Second, firms make decisions on how to operate their power plants. The decisions are whether to have a power plant running or not, and, conditional on running, how much to produce. To make these decisions, firms can use production bids in the centralized market or bilateral production contracts that are arranged ex-ante. Bilateral production contracts account for about one-third of the electricity produced in Spain. They are linked to a particular production unit and specify that the unit will produce a certain amount of electricity during the day. On the contrary, bids in centralized markets do not establish that a unit will produce with certainty; they establish a willingness to produce at different prices, and the final outcomes are resolved in the daily auctions.

This paper takes financial and bilateral production contracts as given. Although I do not endogenize these contracts, I control for them in the empirical analysis. Bilateral production contracts are observed in the data because they need to be communicated to coordinate the operation of the electric system. Financial contracts are not observed, but they have been found to be a crucial factor in determining the optimal bids of the agents (Wolak, 2000; Bushnell, Mansur and Saravia, 2008). I estimate these contracts from the data in the empirical analysis, as in Wolak (2003), Hortaçsu and Puller (2008) and Allcott (2012).

2.1 The day-ahead market

I study the most important auction of the centralized markets: the day-ahead market. Firms in the day-ahead market offer electricity for each hour of the next day. Firms submit their bidding strategies all at once and production for each hour is auctioned simultaneously. Therefore, the day-ahead market is a set of twenty-four simultaneous multi-unit good auctions. Roughly 80% of the electricity allocated in centralized markets is sold through this day-ahead market. Financial contracts are also often indexed at this price, and therefore, the day-ahead market sets a reference for a large amount of the electricity traded in the electricity market.

Focusing on the day-ahead market to understand firm strategic behavior is common in the literature.[6] There are other centralized markets that are potentially important: the market dealing with congestion in the network (restrictions market) and the sequential intra-day markets. Both the congestion market and the sequential markets open after the day-ahead market has cleared. Whereas adjustments in the intra-day markets tend to be small, more substantial changes arise in congested areas where firms enjoy local market power. The study of this additional markets is beyond the scope of this paper.

2.2 Bidding in the market

Generating firms in the day-ahead market bid simultaneously for the 24 hours of the next day. Firms submit bids for each production unit. Traditionally, these units are coal, gas or nuclear generators. Nowadays, there are also renewable energy "aggregators," which pool together wind and solar resources at different locations. Each unit can have both simple and complex bids.

Simple bids are step functions for each generation unit that offer a quantity of electricity (in MWh) at a certain price for a particular hour of the day. Each hourly step function can have up to 25 steps per unit. The price bids need to be positive (or zero) and are capped at 180 €/MWh. Furthermore, price bids need to be monotonically increasing. Each generating unit has its own bid, which implies that the aggregate supply curve of a given firm can have potentially many steps. For example, large companies such as Iberdrola or Endesa can submit aggregate supply functions that have more than 500 hourly steps. In practice, agents do not use all the 25 steps for each unit; generally the bids have at most 10 steps per unit.

Thermal units except for nuclear (coal, gas and oil generators) use complex bids.[7] Complex

[6]See for example Kühn and Machado (2004) for the Spanish electricity market and Borenstein, Bushnell and Wolak (2002) for the Californian electricity market.

[7]Even though any supply unit can use complex bids, other technologies do not appear to make use of them in the data. Hydro, wind and other renewable sources do not have substantial startup costs. Nuclear plants have very high startup costs, but they do not use complex bids because they usually stop at most once or twice per year, planning those events well in advance.

bids complement simple bids and are unique for the whole day. Any unit submitting a complex bid for the whole day still has a simple bid for each hour of the day. Complex bids allow firms to specify a unit-specific minimum revenue requirement characterized by two bidding parameters: a variable and a fixed component.[8] A given unit is called to produce only if the gross revenue obtained by the unit during the whole day covers both the fixed and the variable component of the complex bid. The unit is otherwise removed from the market clearing process, even if its hourly simple bids are lower than the market price during some hours of the day.

When solving for the auction outcome, the market operator uses complex bids as constraints to the simple bids in an iterative fashion. First, optimal quantities and prices are found based on simple bids, crossing demand and supply for each hour of the day independently. Then, the market operator checks that the minimum revenue requirement is satisfied for all units, by comparing their gross revenue with the specified complex bids. If the requirements of some units are not satisfied, they are withdrawn sequentially depending on the magnitude of the violations. The procedure is repeated iteratively until none of the complex bids bind.[9]

2.3 Bidding Data

I construct a new data set using publicly available data from the market and the system operator in Spain (Operador del Mercado Ibérico de Energía, Polo Español (OMEL) and Red Eléctrica de España (REE), respectively). The central piece of the data set are the bidding data from the day-ahead market, which are fully observed and can be mapped to the generating units in the market. I map unit codes to additional data sets that contain characteristics such as type of fuel used, thermal rates, age, and location. These data are coupled with auction outcomes, such as equilibrium prices and winning quantities.[10]

In my empirical analysis, I use data from March 2007 until June 2007.[11] During this period, the two largest firms were Endesa and Iberdrola, with a generation market share of 27% and 21%, respectively. The other bigger firms (Union Fenosa, Hidrocantábrico and Gas Natural, a new entrant)

[8] In practice, firms can also submit unit-specific ramping constraints (speed at which firms can change production levels), although they do not make use of them very frequently. Only around 6% of the units use ramping constraints.

[9] Note that this iterative procedure needs not to be the optimal way to solve the market clearing problem. It also does not guarantee a unique solution, which raises a winner's determination problem. It was chosen due to its simplicity and computational tractability when the market was originally conceived in 1996. Other liberalized markets use alternative algorithms that compute the market clearing in one step, which have been enabled by recent computational advances. For a more detailed description of the algorithm, see the online appendix A.

[10] A more detailed explanation of the data sources can be found in the online appendix B.

[11] The reason to look at this sample is to ensure that the regulatory framework is constant during the period of study. Even though the design of complex bids has not changed since the start of the electricity market, other important institutional details have been changing over time. The regulator introduced some sudden changes in the regulatory framework in March 2006, with the approval of the Royal Decree 03/2006, which affected bidding strategies until February 2007. In July 2007, the Spanish electricity market joined the Portuguese market to form the MIBEL market.

Table 1: Summary statics for the day-ahead market

	Mean	Median	St.dev.	Min	Max	N.Obs.
Hourly market price (€/MWh)	34.1	32.1	9.1	7.0	69.7	2,880
Number of thermal units	78.56	78.00	2.46	73.0	83.0	120
Average simple bid						
First step price (€/MWh)	29.5	1.0	55.1	0.0	180.3	174,731
Other steps price (€/MWh)	67.9	54.3	47.7	0.0	180.3	823,467
First step quantity (MWh)	191	190	126	1	823	174,731
Other steps quantity (MWh)	43	22	66	1	803	823,467
Average complex bid						
Fix component (K€)	38.7	0.0	118.5	0.0	605.0	6,395
Var. component (€/MWh)	51.9	39.2	29.7	0.0	180.3	6,395

Notes: Sample from March to June 2007.

had generation shares between 5 to 11%. Most firms were vertically integrated and participated in distribution (regulated and usually performed by incumbent firms) and retailing.

During this time period, coal was the predominant source of energy (25%), followed by nuclear, natural gas and renewables, each of them with a share of approximately 20%. Hydraulic energy accounted for approximately 10% of the production. Due to limited cross-border transmission capacity, international imports represented a very small fraction of total production, being approximately 3%.

The study of bidding behavior is focused on thermal units, which are the ones using complex bids. In the sample, there are 88 thermal units other than nuclear power plants, which account for most thermal units in the Spanish system that are operating during this period. Nuclear plants, cogeneration plants and new plants that are not operational during this period are excluded. I divide the units in three main categories: coal units, combined cycle gas units and gas and oil units.

Table 1 presents summary statistics of the bidding data and market outcomes. There are 2,880 hour-day observations in the sample, with an average market price of 34.1€/MWh. The average number of units in the market is around 79, which submit on average simple bids with 4.41 steps. The distribution of simple bid prices and quantities depends significantly on the step considered, as explained below. Whenever used, complex bids are on average 38.7K€ for the fixed component and 51.9€ for the marginal component. They are used around two thirds of the times.

2.4 Bidding behavior

Before turning to the model, I illustrate some of the patterns to be explained in the data. The main goal of the discussion is to understand how simple and complex bids translate into *(i)* discrete decisions about using a thermal unit or not (startup decisions), and *(ii)* marginal decisions about how much to produce with a given unit.

2.4.1 How do firms use simple bids?

The usual interpretation of simple bids in a multi-unit auction is that they express a marginal willingness to buy or sell. In the context of electricity markets, they express how much output a firm is willing to produce at different price levels in a given hour. However, if firms have startup costs, simple bids need not to be marginal. In particular, the first step is important to determine whether a unit will run or not in a particular hour. If a unit wins its first step, it means that the unit will be running. Conditional on winning the first step, the other steps of simple bids are marginal, as the unit is already turned on if it sets the price with a step higher than the first one.

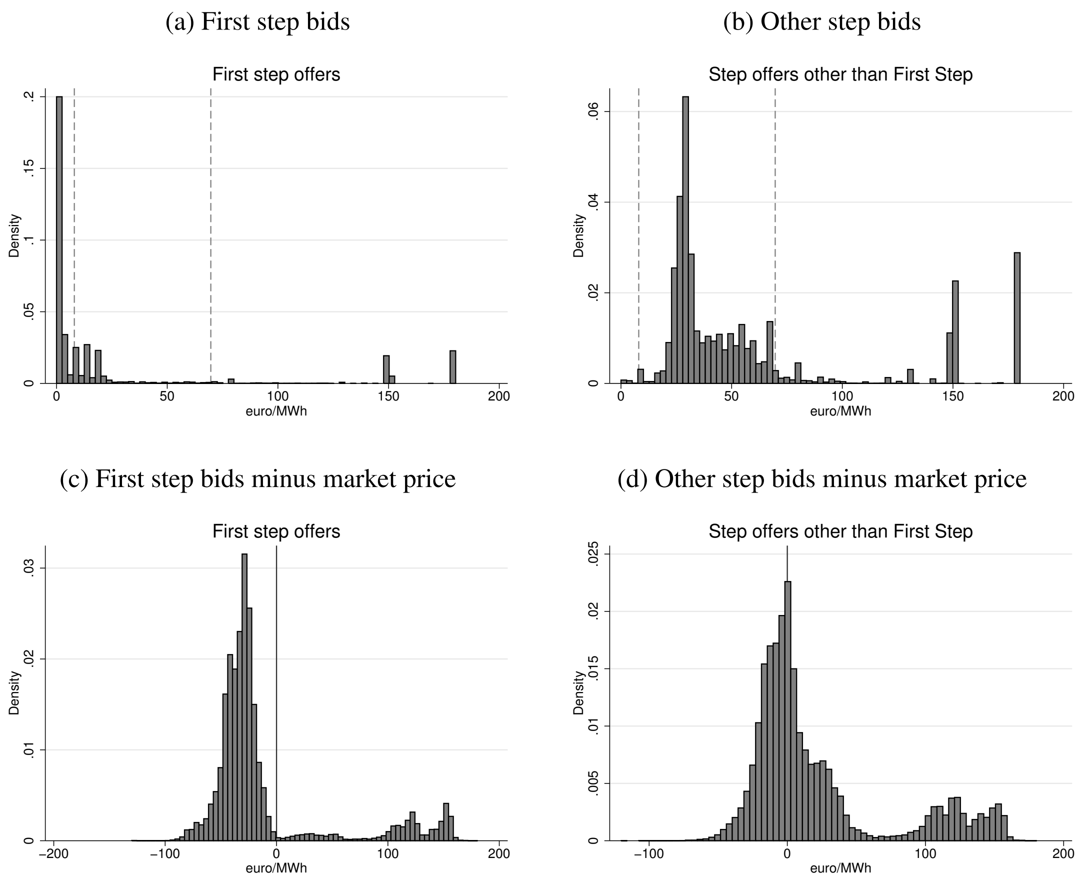
The importance of the first step is very apparent when looking at the bidding data. Firms usually submit a zero bid for the first step in most hours of the day. As seen in Table 1, the median bid is just one Euro, well below any possible marginal cost estimate and the prices observed in the market. This strategy ensures that, conditional on being accepted, the unit will operate for sure. The distribution of bids also shows that, for some hours, firms may submit very high bids instead, ensuring that they will are turned off.

Figure 3 shows graphically the distribution of simple bids for the first step, separated from the rest of the bids. As shown in Figure 3(a), most of first step bids have either very low or very high values, as the first step affects the discrete decision of starting up. In particular, there are important mass points at the price floor (0€/MWh) and around the price cap (180.30€/MWh). Differently, the rest of the steps have a more centered distribution, usually around equilibrium prices observed in the data, highlighting their marginal nature, as seen in Figure 3(b). The same point is conveyed by differencing out the bids from the equilibrium market price, as shown in the histograms 3(c)-(d). Whereas most bids are accumulated either well below or well above the market price for the first step, the rest of the bids are more centered around the market price.

These patterns highlights the discrete decision involved with the first step, which manifests in extreme bids that determine whether a unit will be running or not. Note that this strategy can also be used in the absence of complex bids. In fact, units with neither complex bids nor production contracts have even more extreme bids for the first step, as they use this strategy to determine ex-ante whether they will produce or not, ensuring smooth production patterns.

For the purposes of inferring firms marginal valuations, these patterns point out that the researcher needs to be cautions when inferring marginal costs in the presence of startup costs or other non-convex constraints affecting firms' valuations. Otherwise, the patterns in the first step might look irrational in a paradigm in which firms only have marginal costs and there are no complex bids.

Figure 3: Distribution of simple bids



(a) First step bids

(b) Other step bids

(c) First step bids minus market price

(d) Other step bids minus market price

Note: Sample from March to June 2007. Dashed lines represent minimum and maximum price observed in the sample. One can observe that the distribution of first-step bids appears to be very different from the distribution of "marginal steps." Firms submit either very low or very high first step bids.

2.4.2 How do firms use complex bids?

Complex bids are concerned with the discrete decision about running a unit or not, as they determine whether a unit participates in the market during the day. Firms make frequent use of complex bids. As seen in Table 1, most units use complex bids (66.1%). Alternatively, some firms decide the startup decisions of their units with production contracts, which are set in advance (23.2%). The remainder of units uses the first step to decide the startup, in which case they use extreme bids: very low if they want to ensure production, or very high otherwise.

Conditional on using complex bids, it is useful to have a competitive benchmark in mind when examining the data. As it will be shown below, in a simplified model, it is incentive compatible for a non-strategic firm to set the variable component of their complex bid equal to the marginal cost of inputs, and the fixed component equal to its startup cost. For a strategic firm, these bids will not exactly provide marginal and startup costs, but they will be related.

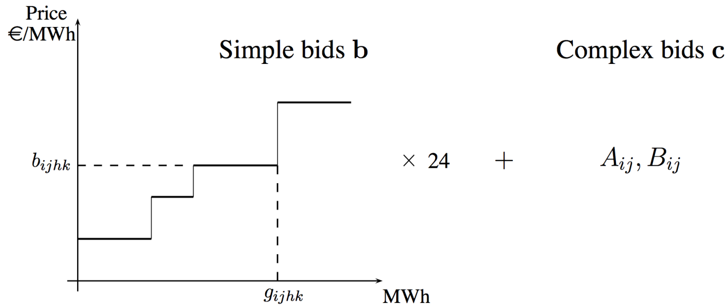
In the data, one can see that these two bid components are indeed correlated with these fundamentals. The size of the minimum revenue requirement varies depending on the type of fuel used. The variable component is on average cheapest for coal units (34.25€/MWh), which have the lowest marginal cost, and most expensive for oil units (94.74€/MWh), which have the highest marginal costs. Regarding the fixed component of complex bids, firms appear to use it only when units are switched off; otherwise they tend to use only the variable component of complex bids. For this reason, the median fixed bid is zero, as seen in Table 1. This is consistent with firms bidding zero when they are already running, and suggests that firms use the fixed component of complex bids to reflect their startup costs in some manner.

3 The model: a multi-unit auction with complex bids

The patterns in the simple bidding data and the frequent use of complex bids make clear the need to account for startup costs in analyzing the bidding process. To formalize firms' decisions, I represent the electricity market as a set of multi-unit auctions with complex bids. There are $i = \{1,...,N\}$ firms, who own a certain number of units that can produce electricity, indexed for each firm by $j = \{1,...,J_i\}$. The goods auctioned in each daily auction are electricity to be produced at each hour of the following day (in MWh).

The units owned by a firm limit the quantity that it can produce. Each unit has a minimum and a maximum capacity, represented by $\underline{q}_{ij}$ and $\overline{q}_{ij}$. Minimum production levels are a feature of thermal power plants. To operate safely, generators need to produce above a certain level. Even though there is some margin of adjustment, to first order this minimum production acts as hard constraint that cannot be relaxed. In the model, if a unit needs to produce within the range $q_{ijh} \in [\underline{q}_{ij},\overline{q}_{ij}]$,

Figure 4: Diagram of bidding strategy for each unit



Firms submit step functions for every unit and every hour of the day. Each unit can also have a fixed and variable complex bid.

whenever it is running.

The model considers the bidding decisions of the firms on a daily basis.[12] Each firm i chooses a bidding strategy to maximize its expected profits, taking the distribution of other firms' bids as given and conditional on a set of common public information and independent private shocks.

The public information set contains the common information shared by the firms, such as publicly available demand forecasts, wind production forecasts and coal and gas input prices. There are several aspects that could generate private information across firms. For example, firms could be uncertain about input cost shocks, about their maintenance strategy or unit unavailabilities, about their hydro storage levels or about bilateral contracts that might be unobserved.

3.1 Bidding space and auction rules

Equilibrium prices and quantities are determined by the auction rules. The auction rules follow the ones in the Spanish electricity market, described in Section 2. All firms submit individual simple bids for each unit to offer their production at a given price, as they would in a uniform auction. In addition, they can also submit complex bids. Complex bids complement simple bids and are also specific to a particular production unit. Figure 4 presents a diagram of the elements of the bidding strategy for each unit in the market.

3.1.1 Simple bids

Define the collection of simple bids by all firms with the array **b**. Simple bids are hour- and unit-specific step functions. They contain price and quantity pairs $\langle b_{ijhk}, g_{ijhk} \rangle$ for each unit j of each firm i, each hour h and each possible step $k = 1,...,\overline{K}$, where $\overline{K}$ is the maximum number of steps

[12] In practice, the decision of starting up often involves more than one day. The theoretical model abstracts from this longer horizon, which I discuss later in the empirical and simulation sections.

and is set by the auctioneer at 25. In practice, this constraint is almost never biding, as firms do not need to use all the allowed steps.[13]

There are some restrictions to simple bids. Price bids cannot be below a price floor (zero) or exceed a price cap (180.30 €/MWh). The quantity bids are also constrained by the capacity of a unit, as otherwise firms would be offering electricity they cannot produce. Price bids need to be weakly increasing with quantity bids.

Definition 1. *Simple bids for a given firm i are defined as:*

$$\mathbf{b}_i = \left\{ \begin{array}{c} (\overrightarrow{b}_{ijh}, \overrightarrow{g}_{ijh}, K_{ijh}) : \dim(\overrightarrow{b}_{ijh}) = \dim(\overrightarrow{g}_{ijh}) = K_{ijh} \in 1,...,25 \quad \forall j,h \\ b_{ijhk} \in [0,180.30], \quad g_{ijhk} \in [\underline{q}_{ij}, \overline{q}_{ij}] \quad \wedge \quad \forall k > 1 : b_{ijhk} > b_{ijhk-1}, g_{ijhk} > g_{ijhk-1} \end{array} \right\}$$

3.1.2 Complex bids

Define the collection of all complex bids with the array **c**. Complex bids contain a fixed and a variable component of the minimum revenue requirement for each unit j of each firm i, represented by A_{ij} and B_{ij} respectively. The variable component is subject to the same price caps as simple bids.

Definition 2. *Complex bids for a given firm i are defined as:*

$$\mathbf{c}_i = \left\{ \begin{array}{c} (A_{ij}, B_{ij}) \quad \forall j \\ A_{ij} \geq 0, B_{ij} \in [0,180.30] \end{array} \right\}$$

A unit needs to recover the fixed and variable components in order to produce, which acts as an effective implicit minimum revenue requirement. The revenue requirement of each unit is constructed as the fixed component plus the variable component times the daily unit output, i.e. $A_{ij} + B_{ij} \sum_{h=1}^{24} q_{ijh}$, where q_{ijh} represents the equilibrium quantity of unit j at hour h.

All the offers of a unit are taken out from the market whenever the minimum revenue requirement over the day is not satisfied, no matter what the simple bids are. A unit j is always discarded whenever at the equilibrium prices,

$$\underbrace{\sum_{h=1}^{24} p_h q_{ijh}}_{\text{Gross Revenue}} \quad < \quad \underbrace{A_{ij} + B_{ij} \sum_{h=1}^{24} q_{ijh}}_{\text{Minimum revenue}},$$

where p_h represents the equilibrium hourly market price. Otherwise the unit is not discarded and

[13] The underutilization of steps is a well-known phenomena previously studied in the multi-unit auction literature (Chapman, McAdams and Paarsch, 2007; Kastl, 2011, 2012).

its bids are considered in the auction.[14]

3.1.3 Market clearing

The market clearing algorithm searches for the set of complex bids that are satisfied. If the minimum revenue requirement is not satisfied, a unit is discarded. The system operator crosses demand and supply using only the simple bids of those units that have not been discarded. The price is determined by the last accepted simple bid, as in a uniform auction.

Let S denote all possible combinations of units being accepted (i.e., having their minimum revenue requirement or complex bid satisfied) and let s denote one of these combinations. As an example, consider a market with two firms, each of them with two units. A possible combination of units being accepted is a set s where units 1 and 2 for firm 1 have their revenue requirement met, but only unit 1 from firm 2 is accepted. Unit 2 is withdrawn because its revenue requirement is not covered. Therefore, s would contain three units in this particular example.

The algorithm initially considers all simple bids to determine equilibrium prices, i.e. it considers the set s_0 in which all units are accepted. It then iteratively discards the simple bids of those units whose minimum revenue requirement is not satisfied, until all remaining units belonging to the equilibrium set s^* cover their revenue requirements.

3.2 The profits of the firm

Given the bidding rules, expected profits of firm i for a given day can be expressed as the sum of expected profits over different combinations of accepted units, i.e.,

$$\mathbb{E}_{-i}[\Pi_i(\mathbf{b},\mathbf{c})] = \sum_{s\in S} P(s|\mathbf{b}_i,\mathbf{c}_i)\mathbb{E}_{-i}\Big[\,\Pi_i(\mathbf{b}_{is},\mathbf{b}_{-is})\Big|s\Big], \tag{1}$$

where $P(s|\mathbf{b}_i,\mathbf{c}_i)$ defines the probability of a set of s of units being accepted, conditional on firm i's bids. Note that this probability depends implicitly on the distribution of beliefs about other firms' strategies, $\{\mathbf{b}_{-i},\mathbf{c}_{-i}\}$.[15] Conditional on a given state s, only the simple bids that are not removed from the supply curve determine market outcomes, which I denote with $\{\mathbf{b}_{is},\mathbf{b}_{-is}\}$. Firm i still remains uncertain about the exact values of $\mathbf{b}_{-is}$, and therefore the expectation is taken over beliefs about other firms' strategies.

[14] In practice, all units are discarded if their minimum revenue requirement is not satisfied, but the converse is not necessarily true. I abstract from this feature of the iterative procedure used in the Spanish electricity, as it simplifies characterizing the problem of the firm. Only in 3% of the cases a unit is taken out from the supply curve, but it could have recovered its minimum revenue requirement. See the online appendix for an extended discussion.

[15] Formally, let $\rho_{nj}(\mathbf{b},\mathbf{c}) = \sum_{h=1}^{24} p_h(\mathbf{b},\mathbf{c})q_{njh}(\mathbf{b},\mathbf{c}) - A_{nj} - B_{nj}\sum_{h=1}^{24} q_{njh}(\mathbf{b},\mathbf{c})$, then

$$P(s|\mathbf{b}_i,\mathbf{c}_i) \equiv \mathbb{E}_{-i}\Big[\prod_{n=1}^{N}\prod_{j=1}^{J_n}\mathbf{1}\{\rho_{nj}(\mathbf{b},\mathbf{c}) \geq 0 \text{ if } j \in s \text{ or } \rho_{nj}(\mathbf{b},\mathbf{c}) < 0 \text{ if } j \notin s\} \mid \mathbf{b}_i,\mathbf{c}_i\Big].$$

The profit function for state s and bids $\{\mathbf{b}_{is},\mathbf{b}_{-is}\}$ is given by,

$$\Pi_i(\mathbf{b}_{is},\mathbf{b}_{-is}) = \left(\sum_{h=1}^{24} p_h(\mathbf{b}_{ihs},\mathbf{b}_{-ihs})(Q_{ih}(\mathbf{b}_{ihs},\mathbf{b}_{-ihs})-\nu_{ih})\right) - \sum_j C_{ij}(\mathbf{q}_{ij}(\mathbf{b}_{is},\mathbf{b}_{-is})), \tag{2}$$

where $p_h(.)$ represents the equilibrium price, $Q_{ih}(.)$ is the total quantity sold by firm i at hour h, ν_{ih} are the financial contracts of the firm, and $C_{ij}(.)$ represents the daily costs of unit j belonging to firm i, which depends on the vector of hourly equilibrium unit quantities. Note that, whereas the hourly market outcomes can be separated on an hourly basis, the cost function is allowed to present time interdependencies.

The profit function is the gross revenue of the firm minus its costs. The gross revenue is stated as the price times the net selling position of the firm (production minus financial contracts), represented by $Q_{ih}(.)-\nu_{ih}$. This is the relevant quantity that determines the incentives of the firm to drive the price either up or down.[16] The actual monetary flow would also include an additional term from the sale of forward contracts, which is sunk at this stage.

When there is market clearing, the net physical quantity allocated to be produced by the firm, represented by $Q_{ih}(.)$, needs to be equal to the residual demand in the market, represented by $D^R_{ih}(.)$. For a particular realization of bids, in equilibrium, supply equals residual demand, i.e. $D^R_{ih}(\mathbf{b}_h) = Q_{ih}(\mathbf{b}_h)$.

Cost structure Complex bids are introduced due to the presence of valuation complementarities at the production level, which arise due to dynamic costs. For this reason, it is important to understand how costs enter the profit function of the firm, as they affect optimality conditions.

I focus my analysis on the cost structure of thermal units (coal, oil and gas), which are the ones that submit complex bids. Expressing the cost function implicitly in terms of equilibrium quantities, the cost function for unit j is expressed as follows,[17]

$$C_{ij}(\mathbf{q}_{ij}) = \sum_{h=1}^{24}\left(\alpha_{ij1}q_{ijh} + \frac{\alpha_{ij2}}{2}\tilde{q}^2_{ijh} + \frac{\alpha_{ij3}}{4}(q_{ijh}-q_{ij,h-1})^2\right) + \beta_{ij}\mathbb{1}^{\text{start}}_{ij},$$

where $\alpha_{ij1},\alpha_{ij2}$ represent unit-specific marginal costs of production, α_{ij3} represents the costs of changing production levels rapidly (also known as ramping costs), $\tilde{q}_{ijh}$ represents the quantity over the minimum production level $\underline{q}_{ij}$, i.e. $\tilde{q}_{ijh} = \max\{q_{ijh}-\underline{q}_{ij},0\}$, and $\mathbb{1}^{start}$ represents a dummy variable that takes the value of one when a unit gets switched on, which implies incurring a startup

[16] This issue has been explored extensively in the literature. See, for example, Wolak (2000) and Bushnell, Mansur and Saravia (2008).

[17] The cost specification parallels Wolak (2007). It is also consistent with engineering models that are frequently used by firms to plan their decisions (Baíllo et al., 2001).

cost β_{ij}.[18]

The dynamic structure comes from the ramping costs (α_3) and the startup costs (β). The problem of starting up becomes non-trivial due to the fact that, whenever units are turned on, they need to produce at least $\underline{q}_{ij}$, which is usually 30% to 40% of their maximum capacity. Therefore, to avoid shutting down at a particular time, a unit needs to keep producing a non-negligible amount of electricity.

3.3 Optimality conditions

The goal of the firm is to choose its bids $\{\mathbf{b}_i, \mathbf{c}_i\}$ to maximize its expected profit. The model above allows to derive optimality conditions at the firm level. In the context of the auction design considered, one needs to characterize the optimal strategies for both simple and complex bids, which constitute the bidding strategy of the firm. Under certain assumptions, first-order conditions can be derived that are amenable for empirical estimation.

For both simple and complex bids, I make the assumption that ties with other firms or with different units within a given firm do not happen with positive probability, which allows me to avoid the problems that arise in the presence of ties (see Kastl, 2011, for a discussion).[19]

3.3.1 Optimality conditions for complex bids

To derive optimality conditions with respect to complex bids, it is useful to note that they only affect the profit function of the firm through the probability of a given set of complex bids being accepted, given by $P(s|\mathbf{b}_i, \mathbf{c}_i)$ in expression (1). Proposition 1 summarizes the main result.

Proposition 1. *Assume $P(s|\mathbf{b}_i, \mathbf{c}_i)$ is differentiable in A_{ij}. If $\frac{\partial P(s|\mathbf{b}_i, \mathbf{c}_i)}{\partial A_{ij}} \neq 0$ for some s, a necessary first-order condition of optimality for A_{ij} is*

$$\mathbb{E}_{-i}\left[\Pi_i^{j\ in}(\mathbf{b},\mathbf{c}) - \Pi_i^{j\ out}(\mathbf{b},\mathbf{c}) \;\Big|\; \sum_{h=1}^{24} p_h q_{ijh} = A_{ij} + B_{ij} \sum_{h=1}^{24} q_{ijh}\right] = 0, \tag{3}$$

where $\Pi_i^{j\ in}$ represents the expected profit of firm i when unit j's is accepted and $\Pi_i^{j\ out}$ represents

[18] Note that I have implicitly assumed that a unit switches on or off at most once every day. This simplification makes the analysis more clear and is consistent with the empirical evidence.

[19] There are several justifications behind this assumption. First, ties are relatively infrequent in the data. I examine bids that are near the market price (5€ band) and count the number of ties as potential ties. Potential ties with other firms happen around 6% of the times. Potential ties at the firm level are relatively infrequent, and happen around 5% of the times. Similar percentages arise using alternative bands. Second, the average size of the marginal step is small (around 30 MWh) compared to the quantities that big firms are selling (usually well above 5,000 MWh), and thus rationing is economically not very important. Finally, in the empirical application, I use a smooth approximation of the first-order conditions in which rationing due to ties does not occur. I discuss the rational for using a smooth approximation in the empirical section.

the expected profit of firm i when unit j is discarded, computed at the point at which the unit's complex bid is marginal.[20]

Proposition 1 states that the firm chooses a complex bid such that the opportunity cost of being accepted versus being rejected are equalized in expectation at the point at which the minimum revenue requirement is just satisfied, i.e., they set the incremental profit of starting up a particular unit for the day equal to its startup costs at the margin.[21] The result follows from observing that marginal changes in complex bids affect outcomes along the range at which the complex bid is just binding, i.e. when $\sum_{h=1}^{24} p_h q_{ijh} = A_{ij} + B_{ij} \sum_{h=1}^{24} q_{ijh}$.

In a competitive environment in which a firm has only one unit and behaves as a price taker, Proposition 1 implies that the optimal complex bid is such that the unit breaks even in expectation, when it is just accepted. For the case in which there are no quadratic and ramping costs ($\alpha_2 = 0, \alpha_3 = 0$), the unit would set the fixed component equal to its startup cost and the variable component equal to its marginal cost, i.e., $A_{ij} = \beta_j$ and $B_{ij} = \alpha_{1j}$. In a strategic environment, however, Equation (3) also captures the fact that the profit of the rest of the units owned by a firm can change depending on whether unit j is accepted or not. If there is a strategic value to withhold capacity and increase equilibrium prices, then a firm will choose complex bids such that the unit still makes positive profits at the point at which it is just accepted.

3.3.2 Optimality conditions for simple bids

To derive first-order conditions for simple bids, I focus on the first-order conditions with respect to the price offers.[22] A necessary condition for simple bids to be consistent with optimality is that there are no profitable local deviations. For example, a firm must be indifferent between raising or lowering the whole bidding offer at a given step of the supply curve, given by a bid b_{ijkh}, for unit j at step k and hour h,

$$\sum_{s \in S} P(s|\mathbf{b}_i, \mathbf{c}_i) \frac{\partial \mathbb{E}_{-i}[\Pi_i(\mathbf{b})|s]}{\partial b_{ijkh}} + \sum_{s \in S} \frac{\partial P(s|\mathbf{b}_i, \mathbf{c}_i)}{\partial b_{ijkh}} \mathbb{E}_{-i}[\Pi_i(\mathbf{b})|s, \frac{\partial P(s|\mathbf{b}_i, \mathbf{c}_i)}{\partial b_{ijkh}} \neq 0] = 0. \quad (4)$$

Whereas the first term is the usual uniform auction optimality condition, the second one arises due to the presence of complex bids.

[20] See the appendix for a derivation. Formally, $\Pi_i^{j \text{ in}} = \sum_{s_{-j} \in S_{-j}} P(s_{-j}|\mathbf{b}_i, \mathbf{c}_i) \mathbb{E}_{-i}[\Pi_i(\mathbf{b}_s, \mathbf{c}_s)|s = \{s_{-j}, j\}]$, and $\Pi_i^{j \text{ out}} = \sum_{s_{-j} \in S_{-j}} P(s_{-j}|\mathbf{b}_i, \mathbf{c}_i) \mathbb{E}_{-i}[\Pi_i(\mathbf{b}_s, \mathbf{c}_s)|s = \{s_{-j}\}]$, where S_{-j} denotes all possible combinations of accepted units excluding j.

[21] In a dynamic context in which different days are interrelated, the difference in profits might include an option value of being either turned on or off for the next day. I discuss this issue in the empirical section.

[22] It is also possible to use quantity offers to derive first-order conditions (Wolak, 2007; Kastl, 2011). I focus on price deviations because the price of the bids is the dimension that firms appear to modify more frequently.

The second term represents the probability that the firm might affect the likelihood of certain units being accepted when setting the market price. Therefore, it is only non-zero when a complex bid is just accepted *and* the unit sets the marginal price. In this setting, the term is likely to be small, as these two events will rarely happen simultaneously. In fact, empirically there are no instances in which these two events happen jointly. Even if these two events occurred jointly, the term tends towards zero as the conditional joint probability of the two events tends to one, due to the envelope condition implied by Proposition 1.

Given these theoretical and empirical findings, to derive the first order condition used in the empirical application, I assume that the first order condition with respect to the bid b_{ijkh} can be reduced to the direct effects of the simple bid on profits, i.e., the first term in (4).[23]

Assumption 1. *Marginal deviations of simple bids at a single step k and hour h are primarily captured by their marginal effects on conditional profits, this is,*

$$\sum_{s\in S}\frac{\partial P(s|\mathbf{b}_i,\mathbf{c}_i)}{\partial b_{ijkh}}\mathbb{E}_{-i}[\Pi_i(\mathbf{b})|s,\frac{\partial P(s|\mathbf{b}_i,\mathbf{c}_i)}{\partial b_{ijkh}}\neq 0]\approx 0.$$

Assumption 1 allows to treat the decision of the firm over simple bids in a similar manner as a set of simultaneous uniform price auctions. Therefore, first order conditions with respect to simple bids closely resemble the ones usually found in a multi-unit auction with a uniform pricing rule. This result is summarized in Proposition 2.

Proposition 2. *Let Assumption 1 hold. A necessary first-order condition for optimality of b_{ijkh}, for a given unit j at hour h and bidding step $k>1$, is given by*

$$b_{ijkh}=\overline{\zeta}_{ijkh}-\frac{\mathbb{E}_{-i}[Q_{ih}-\nu_{ih}|p_h=b_{ijkh}]}{\partial\mathbb{E}_{-i}[Q_{ih}|p_h=b_{ijkh}]/\partial b_{ijkh}},\tag{5}$$

where $\overline{\zeta}_{ijkh}\equiv\frac{\partial\mathbb{E}_{-i}[C_i|p_h=b_{ijkh}]/\partial b_{ijkh}}{\partial\mathbb{E}_{-i}[Q_{ih}|p_h=b_{ijkh}]/\partial b_{ijkh}}$ is a weighted expected average marginal cost when b_{ijkh} sets the price.

Proposition 2 states that the bid is equal to the average marginal cost plus a shading factor or markup. The shading factor is composed by the expected inframarginal quantity produced by the firm when the unit is accepted, divided by its effect on equilibrium quantities, which is equivalent to its effect on the residual demand. The effect of the residual demand on bids captures the impacts of the degree of competition faced by the firm. Intuitively, a more inelastic residual demand drives the markup up. Note that this term is not well defined if there is no residual demand realization at

[23] See the online appendix for a more detailed discussion. I present evidence that shows that the omitted term is empirically small and that omitting it does not appear to rise bias concerns in the main estimation.

which the bid sets the price. Therefore, Proposition 2 is only valid if the bid submitted has some positive probability of being marginal.

This condition is similar to the optimality conditions found in Hortaçsu and Puller (2008) and Allcott (2012). As in the usual setting, the net quantity supplied by the firm determines the sign of the cost markup. For a positive net selling quantity, the firm puts a positive markup to its offer, submitting a bid that is higher than its marginal cost. Forward contracts reduce the selling position of a firm, and therefore reduce markups. Indeed, if the firm is a net buyer in the market, either because it has large forward contracts or because it is also a retailer, the bid will be lower than the marginal cost. One difference in this setup is that the presence of complex bids affects the markup, which is a weighted average over the possible combinations of complex bids being accepted.

4 Estimation

The unknown parameters for firm i can be summarized as follows

$$\theta_i = \{\alpha_i, \beta_i, \gamma_i\},$$

where α_i and β_i are marginal and startup costs respectively, and γ_i represent the parameters that affect the forward position of the firm.

The cornerstone of the estimation are the optimality conditions implied by the multi-unit auction bidding game, presented in Propositions 1 and 2. I discuss first the construction of the empirical analogues of both sets of first-order conditions, as well the intuition behind identification. I then present the results.

4.1 Simple bids Moment Conditions

To estimate the unit-specific costs and the forward contracts, I use a generalized method of moments procedure based on the first-order conditions on simple bids implied by Proposition 2, together with a parametrization of marginal costs and forward contracts. The procedure is analogue to previous studies in the multi-unit auction literature, adapted to the particularities of electricity markets.

The first-order condition in (5) can be re-written as follows:

$$\left(b_{ijkh} - \overline{\zeta}_{ijkh}\right) \frac{\partial \mathbb{E}_{-i}[Q_{ijkh} | p_h = b_{ijkh}]}{\partial b_{ijkh}} + \mathbb{E}_{-i}\left[\left(Q_{ih} - \nu_{ih}\right) \,\middle|\, p_h = b_{ijkh} \right] = 0. \tag{6}$$

To construct the empirical analogue of this first-order condition, one needs to estimate its expectation terms. Similar to Hortaçsu and McAdams (2010) and Kastl (2011), I use a bootstrapping

Table 2: Pseudo-algorithm to Simulate Market Outcomes

For a given firm i and auction day t,
1) Fix bidder i bidding strategies in auction day t.
2) Randomly sample bidding strategies for each firm $j \in I \backslash i$ from a set of N similar days.
3) Clear the market using the complex bidding algorithm (see online appendix A for details).
4) Repeat 2-3 B times to obtain a distribution of market outcomes.

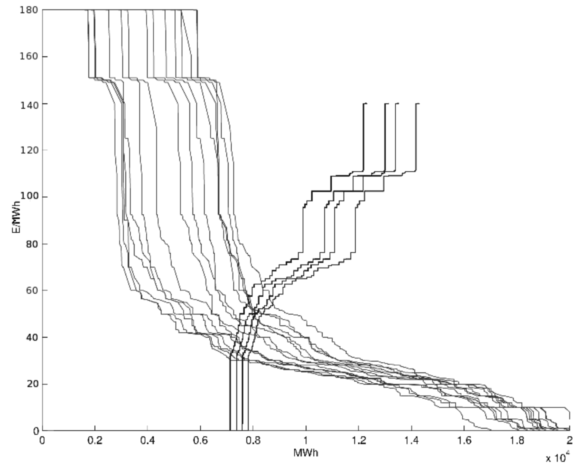
procedure to estimate the expectation over residual demand. The pseudo-algorithm is described in Table 2. The procedure consists in simulating the beliefs of one firm i about the other firms' strategies based on the available data. For a particular firm and day, its strategy is held fix (step 1). For other firms, their strategies are randomly sampled from the data, mimicking the uncertainty that the firm faces (step 2). For a particular draw, and given the auction algorithm, I can compute auction outcomes which determine firm's profit, such as market prices, quantities assigned to each generator and overall quantity sold by the firm (step 3). Repeating this procedure many times, one can approximate expected profits (step 4).

Step 2 is very important as it is meant to mimic firm i's believes about other firms' strategies. I approximate the uncertainty faced by the firm in the market by randomly drawing the strategies of other firms across similar days. Days are classified in four weekday categories (Monday, another weekday, Saturday and Sunday). Within each category, I match similar days by minimizing the squared difference in their maximum demand forecast. For example, to find four similar days for a particular Monday, I take the four Mondays in the sample that have the lowest squared difference between their maximum demand forecast and that of the particular Monday at hand. This group of days will trivially include the same day, together with three other similar days. This selection rule parallels Gans and Wolak (2008), who also pool similar days to construct sample analogues of moment conditions implied by profit maximization.

Figure 5 presents the supply of a given firm and its residual demand for fifteen bootstrapped market outcomes resulting from this procedure. Figure 6 compares the distribution of simulated bootstrapped prices to the one observed during the period of study. One can see that the distribution is matched at the different quantiles of the distribution. This would not be necessarily true if one had matched bids from very different days. Overall, the bootstrapping procedure generates a distribution of prices that matches the one in the sample.

It is important to note that there are three aspects in the bootstrap simulation that are different from the procedure used in other applications. First, the auction is not standard and therefore one

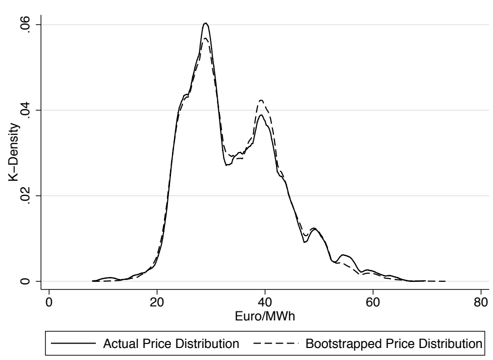
Figure 5: Generating Random Market Outcomes - April 11th 2007, 5pm



Randomly drawing strategies of other players generates a distribution of expected residual demand. Due to complex bids, the ex-post supply curve of a given firm can depend on the particular realization of other firms' strategies.

Figure 6: Distribution of Bootstrapped Market Outcomes

(a) Kernel Distribution of Equilibrium Prices



(b) Summary Statistics

	Price Actual (€/MWh)	Price Bootstrap (€/MWh)	Non-trim. Price BS (€/MWh)
mean	34.16	34.21	34.20
sd	9.07	8.28	8.57
skew.	0.59	0.54	0.58
kurt.	3.37	2.62	2.92
p5	22.72	23.05	22.22
p25	27.48	27.71	27.50
p50	32.18	32.68	32.60
p75	40.01	40.01	40.06
p95	50.16	50.61	52.02
min	7.00	18.00	8.03
max	69.70	65.00	73.40

Notes: Sample of 120 days (24 hours each). Bootstrap samples obtained mixing six similar days. 100 bootstrap samples per day considered. "Price Bootstrap" excludes lower and upper 0.5% of the data.

needs to clear the market using the complex bidding algorithm.[24] Second, due to the presence of complex bids, firms also face uncertainty over their own equilibrium supply curve, as they are not certain about which plants will be asked to turn on (as shown in Figure 5). Finally, in contrast to other auction settings, the population of bidders in the market is held fixed. The randomization comes from mixing similar days for the same population of bidders.

Once market outcomes are simulated, there are still some challenges left to construct the empirical moments, specially to approximate the slope $\mathbb{E}_{-i}[\partial Q_{ijkh}/\partial b_{ijkh}|p_h = b_{ijkh}]$ in (6). Different than other multi-unit auction settings, electricity auctions have limited uncertainty as well as many small steps. In a large auction with many small steps, such as the one considered in this paper, estimating the derivatives at the point at which a bid exactly sets the price can be challenging and numerically unstable, even if many bootstrap samples are drawn.

To address this problem, I follow a smoothing approach that has been used in the context of electricity auctions (Wolak, 2007; Gans and Wolak, 2008). With this approach, both demand and supply are approximated as a continuous function that depends on price. For a given bootstrap sample bs, both the residual demand faced by a firm and its supply curve are approximated as follows,

$$\begin{aligned}\hat{D}^{R,bs}_{ih}(p_h,\mathbf{b}_{-i,h}|s^{bs},bw) &= \sum_{l\neq i}\sum_{j\in s^{bs}_l}\sum_k g_{ljkh}\mathcal{K}\Big(\frac{b_{ljkh}-p^{bs}_h}{bw}\Big),\\ \hat{Q}^{bs}_{ih}(p_h,\mathbf{b}_{i,h}|s^{bs},bw) &= \sum_{j\in s^{bs}}\sum_k g_{ijkh}\mathcal{K}\Big(\frac{b_{ijkh}-p^{bs}_h}{bw}\Big),\end{aligned}$$

where $\mathcal{K}$ is a Kernel cumulative weight and bw is a bandwidth parameter that determines the degree of smoothing.

With this smooth representation, the empirical analog to the first-order condition of unit j at step k in hour h can be constructed. Adding a subscript t to represent the day of the sample, the empirical analogue to the first-order condition is constructed as,

$$m_{ijkht}(\alpha_i,\gamma_i;bw,B) \equiv \frac{1}{B}\sum_{bs=1}^{B}\mathbb{1}(j \text{ in})\widehat{\frac{\partial p_{ht}}{\partial b_{ijkht}}}^{bs}\Big((b_{ijkht}-\bar{\zeta}_{ijkht}(\alpha_i))\widehat{\frac{\partial D^{R,bs}_{iht}}{\partial p_{ht}}}+(Q^{bs}_{ijkht}-\nu_{iht}(\gamma_i))\Big), \tag{7}$$

where B represents the number of bootstrap simulations that are taken for each day in the bootstrapping algorithm. Note that $\frac{\partial p_{ht}}{\partial b_{ijkht}} = \frac{\partial Q_{ih}/\partial b_{ijkh}}{\partial D^R_{ih}/\partial p_h - \partial Q_{ih}/\partial p_h}$ in equilibrium, which can be computed using the smooth approximation (Wolak, 2007). See the appendix for a full derivation.

There are two advantages to using this approach. First, the residual demand becomes a continuous object, and therefore its derivative is easier to approximate. Second, differently than before,

[24] This algorithm is described in the online appendix A.

bids near the simulated equilibrium prices have an impact through the Kernel weights. This fact implies that not only bids that exactly set the price are used for inference, but also those near the observed equilibrium price, which can be useful in auctions with many prices and small bids.

To fully characterize the moments, it remains to specify functional forms for marginal costs and forward contracts. For marginal costs, I specify them following the structural cost function presented in Section 3,

$$\overline{\zeta}_{ijkht}(\alpha_i) \equiv \alpha_{j1}+\alpha_{j2}\tilde{q}_{ijht}+\alpha_{j3}(2q_{ij,h,t}-q_{ij,h-1,t}-q_{ij,h+1,t})+\epsilon_{ijkht}.$$

The error term in the marginal cost specification rationalizes the strategies of the firm in the econometric specification. It can be interpreted as a shock on marginal costs known to the firm as well as optimization or specification error. In the results section, I discuss an alternative specification that allows the coefficients to be proportional to input costs.

The specification for forward contracts assumes that firms hedge a percentage of their expected hourly output,

$$\nu_{ht}(\gamma_i) \equiv \gamma_i q_{ht}+\varepsilon_{ht},$$

where q_{ht} represents the expected quantity sold at the day-ahead market. One justification for this parametric assumption is that it is common in the industry to refer to the amount of hedging as the percent of the output that is hedged. Therefore, I assume that firms target a certain share of financial hedging during the period.

With all these elements, the moments are constructed. Given the potential endogeneity and measurement error of markup terms, I use temperature and the publicly available demand forecast as instruments, represented with Z_{ht}. For a given firm i, the empirical moments that I use are,

$$\sum_{t=1}^{T}\sum_{k=1}^{K} Z'_{ht} m_{ijkht}(\alpha_i,\gamma_i;bw,B) \quad = \quad 0, \forall j,h,$$

which provides a consistent estimate for α_i and γ_i as the number of simulations B and time periods T grow. The estimate will converge at a parametric rate in T as long as one can ensure that the non-parametric components in $m_{ijkht}(\alpha_i,\gamma_i;bw,B)$, which are estimated in a preliminary stage, have converged as $Bbw \to 0$, $B \to \infty$. One needs to assume that the number of similar days required to approximate firms' expectations goes to zero as the sample size goes to infinity, i.e. $N/T \to 0$ as $T \to \infty$.

Note that to construct the moments I average them both over time and across steps. The idea to average the moments across steps is that the choice of steps and their size is potentially endogenous. However, in practice, firms bid using similar number of steps and quantities. Using each step as a separate moment does not appear to affect the results in this application.

Standard errors are constructed using bootstrapping. Due to the temporal nature of the data, I perform block-bootstraps, where the blocks are taken for seven days. A week seems a natural length for the bootstrap blocks in this application, given the seasonality of demand over the week that affects both startup patterns and output in this market.

Identification In the context of electricity markets and using similar first-order conditions, previous work has shown that forward contracts can be identified if marginal costs are observed (Hortaçsu and Puller, 2008), and that marginal costs can be identified if forward contract data is available (Wolak, 2000). I show that identification of both elements can be performed by imposing reasonable restrictions on the cost functions and the forward position.

The key idea behind the identification is to notice that there are two reasonable economic constraints that provide additional degrees of freedom for identification. First, the forward position is a financial position at the firm level that is predetermined at the time of the auction. It is common across the different units, which implies that there are at most 24 forward parameters per day (one per hour). Second, input costs for coal and gas commodities do not fluctuate within a day and marginal costs for power plants tend to be well captured with a finite set of parameters, which implies that one can approximate the cost structure of a power plant reasonably well with a limited number of parameters per day.

Suppose that one could observe firms bidding a very rich strategy, where several units are setting the price with some probability across several hours of the day. If firms have 3 or 4 steps per hour for each unit at the margin, that would generate enough moments to estimate the parameters. The estimation of the forward position would benefit from observing different units at the same hour with different markups (i.e. variation in the inframarginal quantity and the slope of residual demand across units). The variation in bids and markups across hours would help identify the marginal cost for each unit. Indeed, one could identify such parameters very flexibly on a daily basis, by exploiting differences in bids within and across units throughout several hours of the day.

In practice, such high-frequency identification presents challenges. First, for the first-order conditions to be valid, a given unit needs to set the equilibrium price with positive probability. Given the limited degree of uncertainty in these markets, one needs to rely on both daily, weekly and seasonal demand variation, so that the different parts of the supply curve are explored and thus different units are at the margin. Furthermore, to the extent that firms do not exactly optimize their bids at such high frequency, the implied non-parametric estimates could be noisy. In this line, such identification strategy would require to consider moments at the daily level, whereas a more parametric estimation only requires moments to be valid as they are averaged across time. For these reasons, the identification strategy for both marginal costs and forward contracts relies on imposing a structure that is constant over the period of study.

4.2 Complex bids Moment Conditions

As shown in the previous section, first-order conditions with respect to complex bids map into the startup costs. To see this relationship more clearly, one can re-write the first-order condition as follows,

$$\mathbb{E}_{-i}\Big[\tilde{\Pi}_i^{j\ \text{in}}(\alpha_i,\gamma_i) - \tilde{\Pi}_i^{j\ \text{out}}(\alpha_i,\gamma_i)\ \Big|\sum_{h=1}^{24} p_h q_{ijh} = A_{ij} + B_{ij}\sum_{h=1}^{24} q_{ijh}\Big] = \beta_{ij},$$

where $\tilde{\Pi}_i^{j\ \text{in}}, \tilde{\Pi}_i^{j\ \text{out}}$ represent the marginal profit of the firm ignoring startup costs. Representing profits without startup costs allows to note that the firm will startup a unit j as long as the additional net profit from doing so covers its startup costs, β_{ij}.

I use an analogous procedure to the bootstrap method in Table 2 to estimate the elements of this first order condition. Given an estimate of the marginal cost and forward contract parameters, $\{\hat{\alpha}_i, \hat{\gamma}_i\}$, one can construct the sample analog of the left-hand side of the above expression. For each unit j and bootstrap sample bs, I simulate profits both imposing that the unit is accepted (to obtain market outcomes for $\tilde{\Pi}_i^{j\ \text{in}}$) and imposing that the unit is rejected (to obtain market outcomes for $\tilde{\Pi}_i^{j\ \text{out}}$). The residual from the difference in profits from having a unit just in or just out (ignoring startup costs) becomes the estimate of the startup cost itself.

Similar to the simple bidding estimation, empirical observations in which the minimum revenue requirement is just satisfied are rarely observed in practice, even after augmenting the data with the bootstrap procedure. I use a kernel estimator to approximate this term around those demand realizations for which the minimum revenue requirement is close to being satisfied.[25]

The empirical analog to the first-order condition for a unit j and day t is given by,

$$f_{jt}(\beta,\hat{\alpha}_i,\hat{\gamma}_i;bw,B) = \frac{1}{B}\sum_{bs=1}^{B} w_{ijt}^{bw,bs}\cdot\Big(\tilde{\Pi}_{it}^{j\ in,bs}(\hat{\alpha}_i,\hat{\gamma}_i) - \tilde{\Pi}_{it}^{j\ out,bs}(\hat{\alpha}_i,\hat{\gamma}_i) - \beta_j\Big), \tag{8}$$

$$\text{with} \quad w_{ijt}^{bw,bs} = \frac{1}{bs}\kappa\Big(\frac{A_{ijt} + \sum_{h=1}^{24}(B_{ijt} - p_{ht}^{bs})q_{ijht}^{bs}}{bw}\Big),$$

where κ is a probability density function. In my application, I use a Gaussian kernel and I set the bandwidth based on Silverman's optimal rule of thumb.

To create the empirical moments, I use the average over time for each technology and day of

[25] Note that this estimate relies on identifying an object conditional on an event that happens with zero probability and it could be subject to the Borel's paradox, which points out that the object needs not to be continuous when approaching that point. In this application, by inspection the object that is being approximated appears to be smooth around zero and locally linear, thus the locally linear kernel seems an appropriate choice.

the week,

$$\sum_{t\in d_w}\sum_{j\in\tau} f_{jt}(\beta,\hat{\alpha}_i,\hat{\gamma}_i;bw,B) \quad = \quad 0, \forall\tau,d_w,$$

where $\tau = \{Coal,\ CCGT\}$ represents the technology and d_w represents the day of the week. I include an additional moment to control for the slope of the locally linear kernel. I set $bw = 1.06\hat{\sigma}n^{-1/5}$, where $\hat{\sigma}$ is the standard deviation of the net minimum revenue requirement.[26] Standard errors are constructed using the block-bootstrap methodology described in the first step.

Identification The estimator gives an expression for startup costs of those units that use complex bids. Complex bids unveil firms' indifference point for starting up a unit, as they provide a firm's contingent plan. The identification strategy relies on credibly backing out the point at which the minimum revenue requirement is just satisfied, through the bootstrapping mechanism, while correcting for the presence of market power. Startup costs are thus identified as long as startup happens with positive probability, even when only one outcome (on/off) is observed ex-post.

Even with the richness of the bidding data, there are challenges to identifying startup costs. First, in some situations the minimum revenue requirement will be far away from being satisfied, and therefore the bootstrapping technique will not be able to point-wise recover the indifference point. Second, some units do not use complex bids, and therefore this methodology cannot be used. Finally, some units are already on, in which case there is no clear mapping to the startup cost.

To identify the startup cost, I focus my attention on units that are switched off and decide whether to startup or not in a given day using complex bids.The main assumption for identification is that fluctuations in demand affect which units are in or out, and not unobservable shocks. To avoid potential concerns, I explicitly exclude units that are unavailable due to technical problems or maintenance in the estimation, which is observed in the data.

Additionally, the decision horizon of the firm is usually longer than one day. Therefore, if there is a continuation value due to the dynamic nature of the problem, then the estimate also captures the difference in the continuation value of being on or off. Therefore,

$$\hat{\Pi}_i^{j\ in}(\hat{\alpha}_i,\hat{\gamma}_i) - \hat{\Pi}_i^{j\ out}(\hat{\alpha}_i,\hat{\gamma}_i) = \beta_j - \Delta V_i^{j\ in-out}.$$

In the presence of a continuation value, the startup cost estimate will tend to be a lower bound to

[26] The convergence rate for the conditional expectation is $n^{-2/5}$, when $bwB \rightarrow 0$ as $B \rightarrow \infty$. Similar to the first step estimator, if one is willing to assume that the number of similar days required to estimate the conditional expectation consistently goes to zero as T grows, i.e. $N/T \rightarrow 0$, then the convergence is parametric in T.

the actual cost.[27] The intuition behind the above equation is that a unit does not need to recover its startup costs during a single day. Consider for example a competitive unit. The unit might decide to startup if it makes a positive profit during the day, even ignoring the startup cost, if it expects to recover such startup cost in the following days of continuous operation.

To control for the potential continuation value, I use weekday dummies. The idea behind this strategy is that the continuation value of starting up a unit depends on the day of the week. For example, a gas unit starting up on Friday will generally only startup for that given day, as on the weekend there is lower demand for electricity, which implies that the continuation value of that startup can be considered to be approximately zero. The continuation value might still not be zero, as some units do operate continuously over the weekend, specially coal plants. However, the Friday fixed effect can be used as a baseline to identify the startup costs of the units, as this is the day in which the continuation value is arguably lowest.

4.3 Results

This section describes the results of the estimation for the marginal and forward parameters as well as for the startup costs. I estimate the parameters for one of the largest firms in the market, which owns several coal, combined cycle gas and oil units. I focus on this firm for several reasons. First, it is one of the two biggest firms in the market. Second, it is the firm that makes most frequent use of complex bids. Finally, among the two biggest firms, it is the one that owns a richer mix of technologies.[28]

This firm has a thermal generating capacity of around 9,000 MW, most of it composed of combined cycle gas unit (around 5,600 MW) followed by oil and coal units (around 2,050 MW and 1,000 MW respectively). Given that oil units never produce during the sample of study and are always discarded by their complex bids, I do not include them in the estimation as their costs cannot be properly identified. The focus of the paper is on coal and combined cycle gas units, which are the relevant units that use complex bids. I also exclude units that are located in persistently congested areas, as those units are rarely used in the day-ahead market and enjoy local market power, thus facing very different incentives not captured by the model. In total, there are eight thermal units.[29]

[27] Unless there are important strategic effects of keeping a unit off (Fershtman and Pakes, 2009), the option value of having the unit already on is always higher than having to start it up.

[28] The other biggest firm makes limited use of complex bids and also, contrary to this firm, decides a large share of its output and operation decisions through pre-arranged production contracts.

[29] In the online appendix, I include results for the other large firm in the market. The costs that I estimate for the second firm are comparable to those of firm 1.

Table 3: Agreggate Marginal Cost Estimates for Firm 1

	(1)	(2)	(3)	(4)	(5)
Coal (€/MWh)	30.46	29.17	25.52	18.00	25.19
	(0.98)	(0.98)	(1.76)	(6.43)	(1.68)
CCGT (€/MWh)	35.74	34.96	32.83	32.83	29.43
	(2.15)	(2.15)	(3.72)	(3.86)	(3.53)
Coal X q (€/MWh^2)			4.47e-02	2.12e-08	5.94e-02
			(1.77e-02)	(2.47e-02)	(1.75e-02)
CCGT X q (€/MWh^2)			8.89e-03	8.88e-03	2.32e-02
			(5.88e-03)	(6.55e-03)	(9.08e-03)
Coal ramp (€/MWh^2)				4.49e-02	
				(2.85e-02)	
CCGT ramp (€/MWh^2)				1.76e-12	
				(3.05e-03)	
Forward Position (%)	88.47	84.09	85.09	85.10	88.77
	(2.82)	(2.82)	(3.50)	(3.48)	(3.58)
Time Periods	120	120	120	120	120
Moments	210	210	210	210	210

Notes: Sample from March to June 2007. Input variable constructed with European fuel prices of coal, natural gas and oil. Heat rates as provided in reports by the Spanish Ministry of Industry. Estimates computed using a GMM estimator. Bandwidth parameter set to 3€.

4.3.1 Marginal cost and forward estimates

To construct the empirical moments, I take 100 bootstrap draws to simulate market outcomes for each day, with a total of 120 days. Each bootstrap mixes between six similar days. The bandwidth parameter is equal to 3€. The baseline estimation considers one moment per unit and hour, with a total of 210 moments. Because the number of similar days, bandwidth and number of moments are a choice relegated to the econometrician, I perform extensive sensitivity analysis in the robustness section. The results across specifications are similar to those presented here.

Table 3 provides the estimates summarized by fuel type.[30] Focusing on specification (3) with quadratic costs, the results indicate that average coal marginal costs are around 25.52€/MWh, whereas combined cycle marginal costs are around 32.83€/MWh. Combined cycle marginal costs appear to fluctuate more than coal costs over time, which is consistent with the fact that gas prices tend to fluctuate more than coal prices. These estimates are reasonable given input costs during the period. The table also includes the forward position at the firm level, which is estimated to be around 85.09% of the quantity that is sold in the day-ahead market.[31]

[30] In the online appendix, I include additional unit-by-unit results.

[31] Even though 85.09% might appear to be a large share of forwarded quantity, previous studies in markets in which

It is important to note that the specification without quadratic costs presented in (1) differs in the implied forward position of the firms. The specification does not instrument for the endogeneity of the inframarginal quantity. Specification (2), on the other hand, instruments for the markup and gives similar average marginal costs and forward position. The preferred specification is (3), which includes quadratic costs, as they are usually considered when modeling the functioning of these units in engineering terms. For the case of coal units, the magnitudes of the quadratic (α_2) and ramping costs (α_3) are in line with previous estimates in the literature for similar power plants (Wolak, 2007).

Adding ramping costs in specification (4) does not affect the results substantially, although the effect appears to be noisily identified. For example, for gas plants, I find no effect and for coal plants, it appears to take out the effect of quadratic costs. In specification (5), I use information on thermal rates for coal units as well as fuel prices at European markets to control for variation in fuel costs over time. Average marginal costs and forward contract estimates do not change substantially when variation of fuel prices over time is accounted for.

4.3.2 Startup cost estimates

The firm of study actively uses complex bids, which allows me to identify the startup cost for all units. I estimate startup costs using equation (8). To construct a sample of the expected difference in profits at the point at which the minimum revenue requirement is just satisfied, I take 100 bootstrap draws of market outcomes for every day, which enables me to ensure that observations next to the minimum revenue requirement are sampled.

I pool units within thermal groups to perform the moment estimation.The coefficients on the covariates are not unit specific, but specific to the type of fuel. I include a coefficient on the size of the unit, which is an important determinant of startup costs. A substantial part of startup costs are the fuel costs incurred to warm up a generator, which increase with the size of the unit.[32] I include controls for the day of the week, which are different by technology, as a way to capture the variation in the option value of startup due to weekly demand fluctuations.

Results from the startup costs estimation are presented in Table 4, in which I report startup costs for typical unit sizes. In the baseline specification (1), weekday fixed effects are included and Friday is taken as a baseline. Units reportedly unavailable due to maintenance or outages are not included in the baseline specification, given that their complex bids might reflect their unavailability.

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forward data are available document large shares of forward contracting. Wolak (2007) documents an average forward position of 88% in the Australian electricity market. It is also consistent with informal discussions with industry participants, who mentioned that they tend to forward a large share of their expected output.

[32] Startup costs could also potentially depend on the status of other units in the same plant. Unfortunately, there is not enough variation regarding the status of units in the same group to test this hypothesis.

Table 4: Startup Cost Estimates for Firm 1

	(1)	(2)	(3)	(4)	(5)	(6)
Coal (€)						
150.0MW	15,977	12,704	15,383	15,985	16,458	13,766
	(4,523)	(3,583)	(3,344)	(4,524)	(4,505)	(3,772)
350.0MW	28,364	26,442	30,593	28,364	33,581	17,005
	(11,341)	(21,760)	(11,622)	(11,348)	(11,903)	(12,533)
CCGT (€)						
400.0MW	21,967	16,326	29,183	21,997	22,457	22,987
	(20,332)	(18,861)	(18,782)	(20,343)	(20,416)	(20,218)
800.0MW	22,431	-101	40,237	22,425	22,540	25,078
	(23,866)	(26,058)	(25,565)	(23,871)	(23,894)	(23,905)
Input Controls	N	N	Y	N	N	N
Weekday Controls	Y	N	Y	Y	Y	Y
Congested Excluded	Y	Y	Y	N	N	N
Unavailable Excluded	Y	Y	Y	Y	N	N
Already On Excluded	Y	Y	Y	Y	Y	N

Notes: Sample from March to June 2007. Dependent variable is the difference in profits of getting one plant in or out from the market. Estimates computed using a locally linear regression around observations for which the minimum revenue requirement is just satisfied. Regression performed by fuel groups controlling different plant sizes.

Specification (1) shows that coal units have startup costs that are increasing in the capacity of the unit. They range approximately between 16,000€ and 28,000€. These economic estimates seem to be in a reasonable ball park when compared to engineering estimates from previously regulated units.[33] Coal estimates without the weekday controls are somewhat lower, as one would expect, as can be seen in specifications (2). Specification (3) is based on marginal costs that allow for input controls and specification (4) includes congested plants. The inclusion of unavailable units has the expected effect of increasing startup cost estimates, as can be seen in specification (5). If units that are already turned on are considered, the estimated opportunity cost goes down, as seen in (6).

Gas units have startup costs around 22,000€, as shown in column (1). The estimates appear to be similar as a function of the plant size. One possible explanation for this finding is that plants that have a capacity of 800MW are composed of two cycles, and have the option of only starting up 400MW.

The estimates for gas units are more sensitive to the removal of weekday fixed effects or to the inclusion of input controls, as can be seen in columns (2) and (3). Note that they are particularly

[33] Based on data from the Spanish Ministry of Industry (1988), I calculate startup costs ranging from 5,000 to 18,500€.

sensitive for the bigger plant. The reason is that there is only one 800MW plant, and its revenue requirement is often far away from being satisfied. This is specially true in the weekends. The estimates do not appear to be sensitive to the inclusion of congested or unavailable units, as well as to the inclusion of units that are already on, as seen in specifications (4)-(6).

The standard errors for startup costs are relatively tight for coal units, but are large for combined cycle units. The larger standard errors are due to the fact that the startup costs are very sensitive to the first step estimates, which have relatively wide standard errors. Another reason for the imprecise identification is that gas plants do not start very frequently in this period, and are often far away from the point at which their minimum requirement is just satisfied. This is particularly problematic once the block bootstrap is used, as we might be missing those few days in which the units are at the margin. Overall, the point estimates for gas plants need to be interpreted with caution.

4.4 Robustness

The above results rely on a particular choice of the smoothing parameter, the number of moments and the number of similar days that are being mixed. Table 6 in the appendix presents a battery of these robustness checks. Overall, the estimation appears to be robust to the econometric choices.

Smoothing parameter The bandwidth parameter is set at 3€, in line with the literature. In the online appendix E, I provide visual evidence on the degree of smoothing that is achieved. Additionally, I check several different smoothing parameters (ranging from 1€ to 5€). As can be seen in Table 6, the smoothing parameter has a relatively minor effect on the estimates. Marginal costs and forward positions remain very stable within the whole range of smoothing parameters. The only parameter that appears to change somewhat is the startup cost for large coal plants (approximately between 22,000€and 37,000€), but it is still within the confidence intervals of the baseline specification.

Number of moments The baseline estimation considers one moment per unit and hour, with a total of 210 moments. Because firms do not appear to modify their bids for every single hour, but rather in hourly blocks, one could wonder whether the level of aggregation of the first-order conditions could significantly affect the results. To examine this possibility, I try different numbers of moment conditions by aggregating the moments of each unit across blocks of hours. Table 6 explores changing the number of moments. These choices have very minor effects on the estimates, suggesting that the potential optimization error contained in hourly first-order conditions is not significantly biasing the estimates.

Number of similar days An important aspect of the simulation is the mixing of similar days, which allows one to infer what firms would have done in likely market conditions. The econometrician needs to decide how many days to consider and how to pool them. I explore the importance of mixing by changing the number of similar days between two to eight. I find that there is no substantial difference between using four, six or eight similar days. The results are more different if I only use two different days, specially for coal startup costs. With only two similar days, one cannot exploit very well the information contained in complex bids, as with limited variation, few observations might be close the point at which the minimum revenue requirement is just satisfied.

5 Startup Costs and Market Power

The estimates obtained in Section 4 can be used to perform several experiments that analyze the importance of startup costs in electricity markets. In this paper, I focus on the effect of dynamic costs in the ability of firms to exercise unilateral market power.

Before turning to the market power analysis, it is important to note that the estimation is only based on necessary first-order conditions implied by optimal bidding, but these conditions are not sufficient to characterize the full optimal strategy of the firm. To perform policy experiments, it is required to specify a simulation model that, with additional structure, can solve for the optimal strategy of the firm.

In Section 5.1, I extend the model to fully characterize the firm's optimal strategy. In Section 5.2, I assess how good the simulation model predicts market outcomes and firm behavior at the estimated parameters. Finally, in Section 5.3, I conduct policy experiments to assess the interaction of start-up costs and market power analysis.

5.1 Simulation Model

I develop a computational model that solves the best response of a strategic firm, given other firms' strategies, using a mixed integer programming approach that ensures that the global optimum for the firm's best response is found.

The model computes the optimal strategy of the firm to maximize profits, assuming that there is no uncertainty and firms have perfect foresight on other firms' strategies. In the absence of uncertainty, the optimal strategy of the firm is similar to a Cournot strategy, with the added discrete decision over which plants to startup and the introduction of minimum and maximum plant capacities.[34] Therefore, the main decisions are how much quantity to produce at each hour of the day, as

[34] In my original thesis chapter (Reguant, 2011), I show how to extend the computational model to deal with uncertainty as well as to the presence of two strategic firms.

well as which units to use when doing so.

The decision of maintaining a unit started up during the last hour of the day has important implications for costs on the following day. If the plant is kept running, there is no need to incur startup costs. To account for the effects of on/off status across several days, I introduce this dynamic link in the computational model. The decisions of the firm are solved with a finite horizon model looking at five days ahead.

The choice of a finite horizon is reasonable in this application. In the industry, firms use finite horizon models without discount (typically between five to ten days) to make their short-run dynamic decisions. By looking at few days ahead, firms have enough information to assess the relevant trade-offs involved in the startup decision. I choose five days to limit the dimensionality of the problem, but I have evaluated longer lengths and it does not appear to make a substantial impact.

The model of the firm is solved using a combinatorial algorithm that checks all discrete combinations of startup decisions to find the global optimum to the problem of the firm. In the online appendix D, I present a complete characterization of the mathematical program that defines the optimization problem of the firm.

5.2 Model Assessment

I assess the appropriateness of the computational model by computing the firm's optimal strategy given the forward contract, marginal cost and startup cost estimates from Section 4. I compare predicted outcomes in the simulation to those observed in the data.

I find that the model captures well the average price, quantity and startup decisions that are observed in the data. The average price in the data is 34.10€ versus 34.35€ in the replication. The standard deviation on the errors is 3.86€. The thermal quantity produced by the firm is 642MWh per hour on average in the original data, whereas it is 667MWh in the replication. The average number of units on a given hour is 5.09 versus 5.01, respectively.

Figure 7 shows the average daily patterns of four main variables replicated by the model, compared to actual outcomes. Additionally, I simulate market outcomes when I take the lower and upper confidence intervals of marginal costs, startup costs and forward contracts. I take the lower bound on marginal and startup costs paired with the upper bound on forward contracts, to compute the lower bound on costs and strategic market power. I take the opposite approach to obtain an upper bound on costs and strategic behavior.[35] To give a sense of the importance of startup costs, I also plot daily patterns for a model in which startup costs are ignored.

[35] These bounds on parameter estimates do not necessarily provide upper and lower bounds on market outcomes, such as market prices, as startup costs can have non-linear effects on startup patterns.

Panel 7(a) shows the evolution of prices over the day, in black. The original data is plotted with a gray solid line. The model predicts accurately the evolution of average daily prices. The daily quantity patterns are also well captured by the model, as seen in panel 7(b). When looking at the quantity patterns, the differences between the model with startup costs (solid black line) and that without startup costs is more apparent (dashed line). Ignoring startup costs overstates the ability of the firm to accommodate fluctuations in daily output.

To show the effects of startup costs in more detail, panels 7(c) and 7(d) show the number of units that are operating during the day on average and the total average capacity that is operating, respectively. The model including startup costs does a good job at predicting that firms do not switch on and off their units frequently, in spite of not capturing its timing exactly.[36] On the contrary, ignoring startup costs, one would predict that firms turn on and off units much more frequently than they actually do, overstating their production flexibility.

Computational model and startup cost estimation With the computational model, it is possible to simulate the firm's optimal best response at given parameter values. As an additional check, one can re-estimate startup costs based on the moments in Figure 7, using a simulated method of moments (SMM). These estimates can be compared to the estimates relying only on necessary first-order conditions implied by optimal bidding.

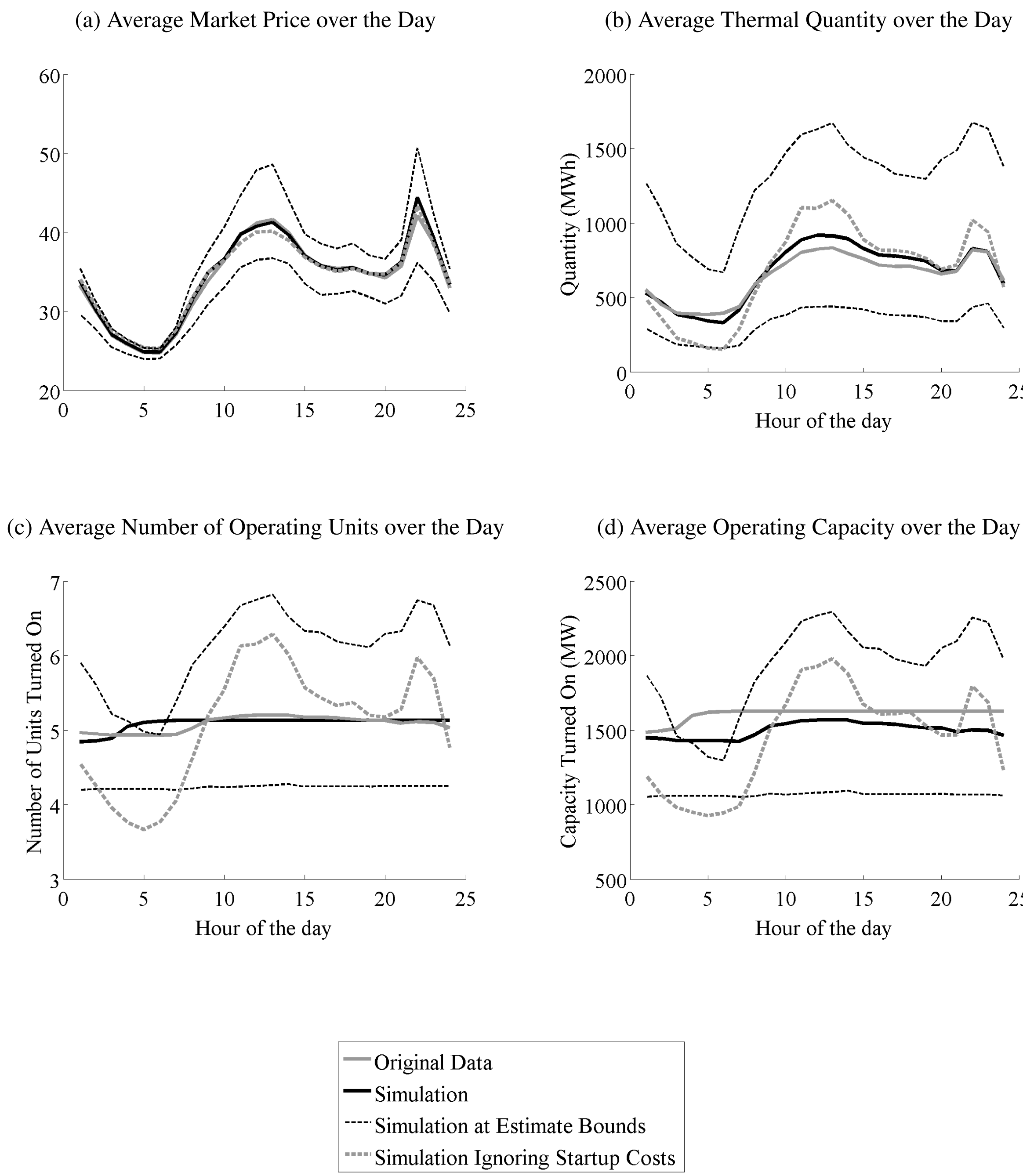
Whereas the first-order conditions are derived from theory, the estimation using a SMM approach is more dependent on how the computational model is specified, as well as how the simulated moments are chosen. It can also be computationally burdensome, as one needs to simulate market outcomes throughout the period for each possible guess of startup costs. As an advantage, the computational model is more explicit about continuation values. It is also feasible even in the absence of auction data, which allows to estimate startup costs in markets where complementary bidding mechanisms might not be in place or bidding data might not be available.

I perform a SMM estimation restricting the dimensionality of the parameters to one startup parameter for coal units and one startup parameter for gas units, using a finite grid in a range of 0 to 80,000 €, in 5,000€ intervals. As moments, I use the ones presented in Figure 7: hourly average price, hourly average quantity, hourly average number of units on, and hourly average capacity turned on.

The sum of squared errors is minimized when coal startup costs are between 5,000 and 15,000€ and gas costs are between 30,000 and 40,000€. Coal estimates appear to be sensitive to the moments used, and for some cases they range from 45,000 to 65,000€, highlighting that only using

[36]One possible explanation for why firms appear to startup before than the model predicts is that, in reality, firms cannot produce at their minimum output level immediately, but need to warm up progressively. Introducing this level of detail in the operations of these plants is beyond the scope of this paper.

Figure 7: Hourly Patterns from Replication Model



data from actual startup patterns can make the identification of startup costs more sensitive in practice. This can be particularly true for coal units, which do not startup and shutdown very often.

The estimates are broadly in the ballpark of those found in the main estimation section. None of the combinations of moments that I consider are maximized with zero startup costs for either technology, emphasizing again the importance of startup costs for understanding firm behavior.

5.3 Market Power Assessment

After evaluating the computational model, I use it to perform counterfactual experiments and assess the degree of market power in this industry. I focus on understanding how accounting for startup costs can affect market power calculations. Traditionally, market power calculations are performed by comparing actual market outcomes to a competitive counterfactual without startup costs (Borenstein, Bushnell and Wolak, 2002; Bushnell, Mansur and Saravia, 2008). Mansur (2008) points out that ignoring dynamic costs could generate important biases in predicted welfare effects of market power.

To assess whether the methods presented in this paper can help reduce these biases, I compute market power estimates comparing the simulated market outcomes to a competitive counterfactual with and without startup costs. The strategic model solves for the firm's best response, as described in section 5.1. The competitive counterfactual is computed by minimizing firm production costs, instead of maximizing profits, also taking other firms' behavior as given. Therefore, it can be interpreted as the firm's first best counterfactual strategy.[37]

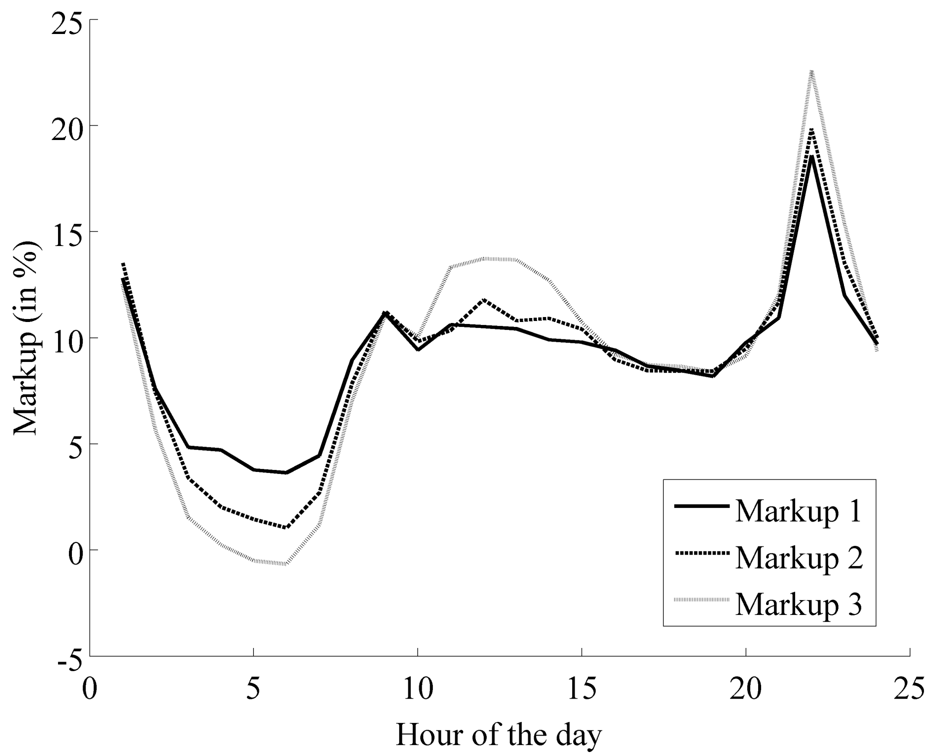
I define a measure of unilateral market power by comparing predicted prices in the strategic simulation ($p^{strategic}$) and predicted prices in the competitive simulation ($p^{competitive}$). At a given day and hour, strategic markups are computed as,

$$Markup = 100 \times \frac{p^{strategic} - p^{competitive}}{p^{competitive}}.$$

I focus on three main experiments. The first experiment computes markups when both the strategic firm and the competitive counterfactual have dynamic costs (Markup 1). The second experiment computes markups when both the strategic firm and the competitive counterfactual do not have any dynamic costs (Markup 2). Finally, the third experiment computes markups when the strategic firm has dynamic costs but these are not accounted for in the competitive counterfactual (Markup 3), which is what has been usually considered in the literature. All three markups can be interpreted as short-run unilateral market power estimates; the only differences arise from whether startup costs are taken into account or not.

[37] The mathematical program for the competitive case is also described in the online appendix D.

Figure 8: Markups and Startup Costs



The graph represents average markups over the twenty-four hours of the day over alternative counterfactuals, depending on whether startup costs are accounted for or not. Markup 1 considers strategic markups when both the strategic and competitive benchmark account for dynamic costs. Markup 2 considers markups when no benchmark has dynamic costs. Markup 3 considers strategic markups when the strategic firm has dynamic costs, but the competitive benchmark does not account for them.

Figure 8 and Table 5 present the results from these experiments. As shown in Figure 8, the counterfactual that compares actual firm behavior with a competitive counterfactual without startup costs generates the most volatile markups (Markup 3). In fact, I find average negative markups at night, which is consistent with previous evidence using a static competitive framework (Bushnell, Mansur and Saravia, 2008). Negative markups are generated because the competitive counterfactual without dynamic costs cannot explain why strategic firms keep producing at night, even when prices drop substantially. These markups are significantly different than those predicted by a model with dynamic costs (Markup 1), as shown in Table 5, column 5.

Removing startup costs from both the strategic and competitive simulations reduces part of this bias, as it does not create such strong asymmetry between the strategic and competitive counterfactual, generating positive markups throughout the day (Markup 2). Predicted markups are still significantly smaller at night, as shown in Table 5, column 4. Furthermore, firm behavior in the absence of startup costs does not fit well actual realized outcomes, as shown in Section 5.2. Finally,

Table 5: Hourly Markups Across Models

	Markup 1	Markup 2	Markup 3	Δ_{2-1}	Δ_{3-1}
Hourly Block 1	8.41%	8.13%	6.57%	-0.29%	-1.85%
	(0.53%)	(0.59%)	(0.62%)	(0.32%)	(0.26%)
Hourly Block 2	4.04%	1.50%	-0.31%	-2.54%	-4.35%
	(0.29%)	(0.17%)	(0.19%)	(0.28%)	(0.28%)
Hourly Block 3	8.18%	7.28%	6.43%	-0.90%	-1.75%
	(0.70%)	(0.82%)	(0.84%)	(0.38%)	(0.38%)
Hourly Block 4	10.19%	10.66%	12.36%	0.47%	2.18%
	(0.48%)	(0.50%)	(0.55%)	(0.36%)	(0.30%)
Hourly Block 5	10.04%	10.71%	12.37%	0.67%	2.33%
	(0.42%)	(0.45%)	(0.49%)	(0.33%)	(0.27%)
Hourly Block 6	8.85%	8.62%	8.87%	-0.23%	0.03%
	(0.39%)	(0.41%)	(0.46%)	(0.22%)	(0.24%)
Hourly Block 7	9.63%	9.86%	9.85%	0.22%	0.22%
	(0.44%)	(0.47%)	(0.51%)	(0.26%)	(0.24%)
Hourly Block 8	13.43%	14.46%	15.77%	1.03%	2.35%
	(1.02%)	(1.07%)	(1.11%)	(0.44%)	(0.39%)

Notes: Sample from March to June 2007. Standard errors in parenthesis. Markup 1 considers strategic markups when both the strategic and competitive benchmark account for dynamic costs. Markup 2 considers markups when no benchmark has dynamic costs. Markup 3 considers strategic markups when the strategic firm has dynamic costs, but the competitive benchmark does not account for them. Each block contains three hours, starting at midnight-3am.

accounting for startup costs in both the strategic and the competitive counterfactual produces the smoothest markups (Markup 1).

By examining Markup 2 and Markup 3, it is possible to compute differences in strategic markups due to dynamic costs, holding the competitive counterfactual constant.[38] Ignoring startup costs (Markup 2) overestimates the ability of the firm to price discriminate across hours with different competitive conditions. In the static model, a firm can freely reduce its output when demand is low, avoiding some of the price fall at night. On the contrary, when demand is high, the static firm is more responsive and increases its output relatively more, reducing price increases. This implies that startup costs exacerbate price fluctuations in this market, making price differences between night and day even more stark.

In sum, these findings highlight that dynamic costs limit the ability of the firm to adjust output. Through the lens of a model without startup costs, firms in the data would appear to have "too many" plants running at night, but "too few" during the day, exacerbating price fluctuations. This would lead to overestimate the amount of market power exercised during the day, and underestimate market power at night, resulting in high markup volatility. Introducing startup costs contributes to explaining why firms do not adjust quantity to high frequency changes in market conditions, producing smoother estimated markups.

6 Conclusions

In this paper, I study a complementary bidding mechanism that is used in electricity markets to allow firms to reflect their dynamic costs of production. I also examine how these dynamic costs affect firm behavior and its incentives and ability to exercise market power.

I extend previous results from the multi-unit auction literature to account for these augmented forms of bidding and derive first-order conditions that can be used in the estimation. I develop a new method to use the information in complementary bids to identify startup costs. I also show that accounting for cost complementarities helps better rationalize the observed patterns in the bidding data.

Finally, I present counterfactual simulations to measure the interaction of startup costs and unilateral market power. I find that the introduction of startup costs generates smoother markups, thus providing a natural correction for markup calculations, which had been previously shown to exhibit downward bias at night. This is explained by startup costs limiting the ability of a strategic firm to change production across hours with different demand conditions, exacerbating price volatility in this market.

[38]The competitive counterfactual used to compute these markups does not have dynamic costs. Relative comparisons would be similar if the dynamic competitive counterfactual were used to normalize markups.

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A Appendix

A.1 Derivation of Proposition 1

Complex bids only affect directly the probability of each complex condition being binding, given by $P(s|\mathbf{b}_i,\mathbf{c}_i)$. Separating the different uncertainty realizations in those in which the minimum requirement is satisfied and those at which the minimum requirement is not satisfied, marginal changes in complex bids only affect outcomes at those states in which the complex bid is just binding, given by $\sum_{h=1}^{24} p_h q_{ijh} = A_{ij} + B_{ij} \sum_{h=1}^{24} q_{ijh}$.

We can partition the state space in states in which unit j is accepted or not, as follows

$$\sum_{s_{-j}\in S} P(s_{-j},j\ in|\mathbf{b}_i,\mathbf{c}_i)\mathbb{E}_{-i}[\Pi_i(\mathbf{b},\mathbf{c}|s_{-j},j\ in)] + P(s_{-j},j\ out|\mathbf{b}_i,\mathbf{c}_i)\mathbb{E}_{-i}[\Pi_i(\mathbf{b},\mathbf{c}|s_{-j},j\ out)] = 0.$$

Omitting $\mathbf{b}_i,\mathbf{c}_i$ in P to ease notation (all probabilities and expectations are conditioned on own bids), the first-order condition becomes,

$$\sum_{s_{-j}\in S} \frac{\partial P(s_{-j},j\ in)}{\partial A_{ij}} \mathbb{E}_{-i}[\Pi_i(\mathbf{b},\mathbf{c}|s_{-j},j\ in) - \Pi_i(\mathbf{b},\mathbf{c}|s_{-j},j\ out)] = 0,$$

by noting that $P(s_{-j},j\ out) = P(s_{-j}) - P(s_{-j},j\ in)$, and that $P(s_{-j})$ does not depend on A_{ij} at the margin, under the assumption of no ties.

Define $\rho_{ij} \equiv \sum_{h=1}^{24} p_h q_{ijh} - A_{ij} - \sum_{h=1}^{24} B_{ij} q_{ijh}$. We can express these probabilities as,

$$P(s_{-j},j\ in) \quad = \quad F(\rho_{ij} \geq 0|s_{-j})P(s_{-j}).$$

Differentiating with respect to A_{ij}, the derivative becomes,

$$\sum_{s_{-j}\in S} f(\rho_{ij} = 0|s_{-j})P(s_{-j})\mathbb{E}_{-i}[\Pi_i(\mathbf{b},\mathbf{c}|s_{-j},j\ in) - \Pi_i(\mathbf{b},\mathbf{c}|s_{-j},j\ out)] = 0.$$

Dividing by $\sum_{s_{-j}\in S} f(\rho_{ij} = 0|s_{-j})P(s_{-j})$, this expression gives the expected difference in profits when the minimum revenue requirement is just satisfied,

$$\mathbb{E}_{-i}\Big[\Pi_i^{j\ \text{in}}(\mathbf{b},\mathbf{c}) - \Pi_i^{j\ \text{out}}(\mathbf{b},\mathbf{c}) \ \Big|\ \rho_{ij} = 0\Big] = 0.$$

A.2 Derivation of Proposition 2

Under assumption 1, we need to show that,

$$\sum_{s\in S} P(s|\mathbf{b}_i,\mathbf{c}_i) \frac{\partial \mathbb{E}_{-i}[\sum_h p_h(\mathbf{b}_h)(Q_{ih}(\mathbf{b}_h) - \nu_{ih}) - C_i(\mathbf{b})|s]}{\partial b_{ijkh}} = 0,$$

implies,

$$b_{ijkh} = \overline{\zeta}_{ijkh} - \frac{\mathbb{E}_{-i}[Q_{ih}(\mathbf{b}) - \nu_{ih}|j\ in, p_h = b_{ijkh}]}{\partial \mathbb{E}_{-i}[Q_{ih}(\mathbf{b})|j\ in, p_h = b_{ijkh}]/\partial b_{ijkh}},$$

where $\overline{\zeta}_{ijkh}$ equals the average marginal cost at the step for the case of constant marginal costs at the step or constant slope of residual demand at p_h. Otherwise, the term is equal to a weighted average marginal cost at the step, defined by,

$$\overline{\zeta}_{ijkh} \equiv \frac{\partial \mathbb{E}_{-i}[C_i(\mathbf{b})|j\ in, p_h = b_{ijkh}]/\partial b_{ijkh}}{\partial \mathbb{E}_{-i}[Q_{ih}(\mathbf{b})|j\ in, p_h = b_{ijkh}]/\partial b_{ijkh}}.$$

Note that the bid only affects market outcomes on a particular hour, therefore one can compute the derivative for every single hour. The only links across hours will be contained in the marginal cost $\overline{\zeta}_{ijkh}$, which depends on expected market outcomes at contiguous hours.

Following Kastl (2011), and noting that the bid only affects market outcomes when it sets the price with positive probability, one can differentiate expected profits with respect to the bids at every hour, which gives

$$\sum_{s \in S|j\ in} P(s|\mathbf{b}_i, \mathbf{c}_i)\Big(\mathbb{E}_{-i}[Q_{ih} - \nu_{ih}|s, p_h = b_{ijkh}] + b_{ijkh}\frac{\partial \mathbb{E}_{-i}[Q_{ih}|s, p_h = b_{ijkh}]}{\partial b_{ijkh}} - \frac{\partial \mathbb{E}_{-i}[C_i|s, p_h = b_{ijkh}]}{\partial b_{ijkh}}\Big) = 0.$$

Note that this derivation uses the fact that $\frac{\partial p_h}{\partial b_{ijkh}} = 1$ when $p_h = b_{ijkh}$ and zero otherwise. Re-arranging the terms gives the above result.

A.3 Derivation of Smoothed Proposition 2

To derive the smoothed out probability, I follow the best-response bidding approach in Wolak (2003). By differentiating the profit function with respect to the bids, one finds

$$\sum_{s \in S|j\ in} P(s|\mathbf{b}_i, \mathbf{c}_i)\mathbb{E}_{-i}[(Q^S_{ih} - \nu_h)\frac{\partial p_h}{\partial b_{ijkh}} + p_h \frac{\partial D^R_h}{\partial p_h}\frac{\partial p_h}{\partial b_{ijkh}} - C'_i(\frac{\partial Q_{ih}}{\partial b_{ijkh}} + \frac{\partial Q_{ih}}{\partial p_h}\frac{\partial p_h}{\partial b_{ijkh}})] =$$
$$\sum_{s \in S|j\ in} P(s|\mathbf{b}_i, \mathbf{c}_i)\mathbb{E}_{-i}[\frac{\partial p_h}{\partial b_{ijkh}}\Big((Q^S_{ijkh} - \nu_h) + (b_{ijkh} - C'_i)\frac{\partial D^R_h}{\partial p_h}\Big)].$$

Note that $\frac{\partial p_h}{\partial b_{ijkh}} = \frac{\partial Q_{ih}/\partial b_{ijkh}}{\partial D^R_{ih}/\partial p_h - \partial Q_{ih}/\partial p_h}$ using the market clearing condition, $Q_{ih} = D^R_{ih}$.

The equation is very closely related to the bidding equation. In the step function setting, whenever the bid sets the price, $\frac{\partial p_h}{\partial b_{ijkh}} = 1$, which brings back the original expression conditional on the bid setting the price ($p_h = b_{ijkh}$).

Table 6: Robustness checks - Cost Estimates for Firm 1

Sim days	bw	# Mom	Coal ($\overline{\alpha}_1$)	CCGT ($\overline{\alpha}_1$)	Startup Coal ($\overline{\beta}_{150}$,$\overline{\beta}_{350}$)		Startup Gas ($\overline{\beta}$)	Forwards (%)
8	3	66	25.10 (2.14)	32.02 (4.86)	17,962 (4,211)	26,442 (11,533)	29,859 (26,505)	83.93 (5.01)
8	3	82	26.06 (1.81)	31.98 (4.52)	14,588 (3,893)	26,666 (9,950)	30,096 (25,049)	83.76 (4.48)
8	3	114	25.65 (1.74)	32.05 (4.13)	15,745 (3,974)	26,714 (9,279)	29,727 (23,139)	83.69 (3.97)
8	3	210	25.48 (1.74)	32.76 (3.86)	15,394 (3,874)	29,692 (9,717)	26,212 (21,924)	84.86 (3.68)
8	1	210	2.55 (1.97)	33.20 (4.81)	12,022 (4,341)	39,627 (9,734)	24,102 (26,198)	86.17 (5.12)
8	2	210	2.53 (1.90)	33.05 (4.33)	14,427 (4,338)	34,189 (9,505)	24,758 (23,924)	85.29 (4.48)
8	4	210	2.62 (1.59)	32.46 (3.48)	14,998 (3,444)	24,907 (9,863)	27,692 (20,328)	84.63 (2.88)
8	5	210	2.66 (1.47)	32.14 (3.28)	14,214 (3,201)	23,693 (9,679)	29,429 (19,565)	84.97 (2.24)
6	3	66	25.24 (2.11)	32.41 (4.71)	18,661 (4,677)	24,419 (12,747)	24,143 (25,884)	84.59 (4.75)
6	3	82	26.21 (1.83)	32.31 (4.39)	15,127 (4,428)	24,675 (11,410)	24,650 (24,616)	84.36 (4.24)
6	3	114	25.73 (1.73)	32.25 (3.97)	16,415 (4,526)	24,878 (10,952)	24,885 (22,616)	84.06 (3.74)
6	3	210	25.52 (1.76)	32.83 (3.72)	15,977 (4,523)	28,364 (11,341)	22,114 (21,529)	85.09 (3.50)
6	1	210	2.56 (1.94)	33.61 (4.61)	12,478 (4,828)	36,995 (10,772)	18,365 (25,306)	86.41 (4.93)
6	2	210	2.54 (1.90)	33.14 (4.26)	14,769 (4,903)	32,567 (11,154)	20,594 (23,881)	85.40 (4.35)
6	4	210	2.62 (1.61)	32.46 (3.40)	15,664 (4,134)	23,519 (11,410)	23,995 (20,094)	84.81 (2.78)
6	5	210	2.66 (1.50)	32.17 (3.19)	14,887 (3,899)	22,074 (11,261)	25,573 (19,169)	85.14 (2.20)
4	3	66	25.48 (1.94)	32.00 (4.92)	17,095 (4,480)	19,617 (10,955)	25,527 (25,605)	84.06 (4.71)
4	3	82	26.39 (1.66)	31.92 (4.59)	13,804 (4,412)	19,837 (9,267)	25,926 (24,192)	83.84 (4.14)
4	3	114	25.97 (1.55)	31.88 (4.23)	14,973 (4,345)	19,968 (8,998)	26,065 (22,635)	83.64 (3.67)
4	3	210	26.23 (1.54)	32.15 (4.05)	14,209 (4,226)	19,809 (9,393)	24,805 (21,880)	84.05 (3.47)
4	1	210	2.60 (1.71)	33.39 (4.75)	11,239 (4,221)	30,501 (10,044)	18,963 (24,641)	86.13 (4.68)
4	2	210	2.59 (1.69)	32.76 (4.57)	13,218 (4,396)	25,772 (9,873)	21,884 (23,945)	84.86 (4.30)
4	4	210	2.65 (1.43)	32.43 (3.56)	14,054 (4,012)	18,144 (9,362)	23,547 (19,978)	84.73 (2.70)
4	5	210	2.69 (1.35)	32.22 (3.28)	13,361 (3,884)	16,865 (9,244)	24,775 (18,932)	85.18 (2.06)
2	3	66	25.67 (2.05)	33.40 (4.87)	14,501 (4,575)	9,652 (14,763)	23,459 (28,092)	84.95 (4.94)
2	3	82	26.40 (1.75)	33.29 (4.51)	11,894 (4,633)	9,862 (12,466)	24,028 (26,464)	84.82 (4.30)
2	3	114	26.07 (1.65)	33.44 (4.24)	12,904 (4,493)	9,798 (12,767)	23,237 (25,211)	84.89 (3.93)
2	3	210	26.19 (1.59)	33.80 (4.00)	12,675 (4,430)	10,028 (12,824)	21,441 (24,082)	85.44 (3.63)
2	1	210	2.63 (1.75)	34.30 (4.88)	10,127 (4,300)	17,190 (12,554)	18,998 (27,725)	86.65 (4.81)
2	2	210	2.60 (1.71)	34.18 (4.42)	12,346 (4,665)	13,281 (12,337)	19,518 (25,713)	85.87 (4.33)
2	4	210	2.67 (1.52)	33.81 (3.56)	11,969 (4,176)	8,211 (12,782)	21,487 (22,258)	85.88 (2.89)
2	5	210	2.72 (1.47)	33.36 (3.37)	10,872 (4,036)	7,143 (12,365)	23,876 (21,629)	86.10 (2.31)

Notes: Sample from March to June 2007. Input variable constructed with European fuel prices of coal, natural gas and oil. Heat rates as provided in reports by the Spanish Ministry of Industry. Estimates computed using a GMM estimator.